CHAMBER OF MINES OF SOUTH AFRICA

EXAMINATION PAPER

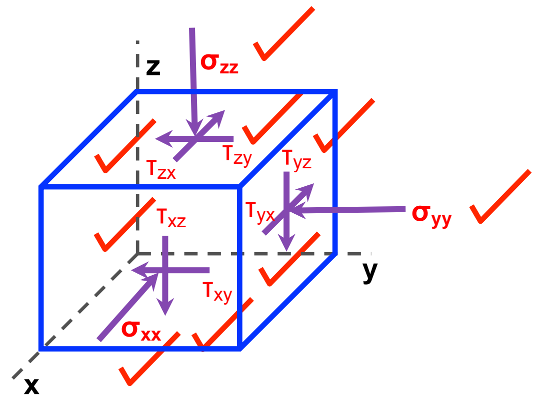
SUBJECT: CERTIFICATE IN ROCK MECHANICS PAPER 1 : THEORY **SUBJECT CODE:** COMRMC1 **EXAMINATION DATE:** MAY 2018 **TIME**: 3 HOURS	**EXAMINER:** WM BESTER **MODERATOR:** H YILMAZ **TOTAL MARKS:** [100] **PASS MARK:** (60%)

QUESTION 1

1.1 Given the stress tensor in x y z Cartesian coordinates below:

$$\begin{pmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{pmatrix}$$

(a) Plot each of these nine stress components on a cubic element. (9)



(b) List the required relationships between pairs of shear stress components for equilibrium to be maintained. (3)

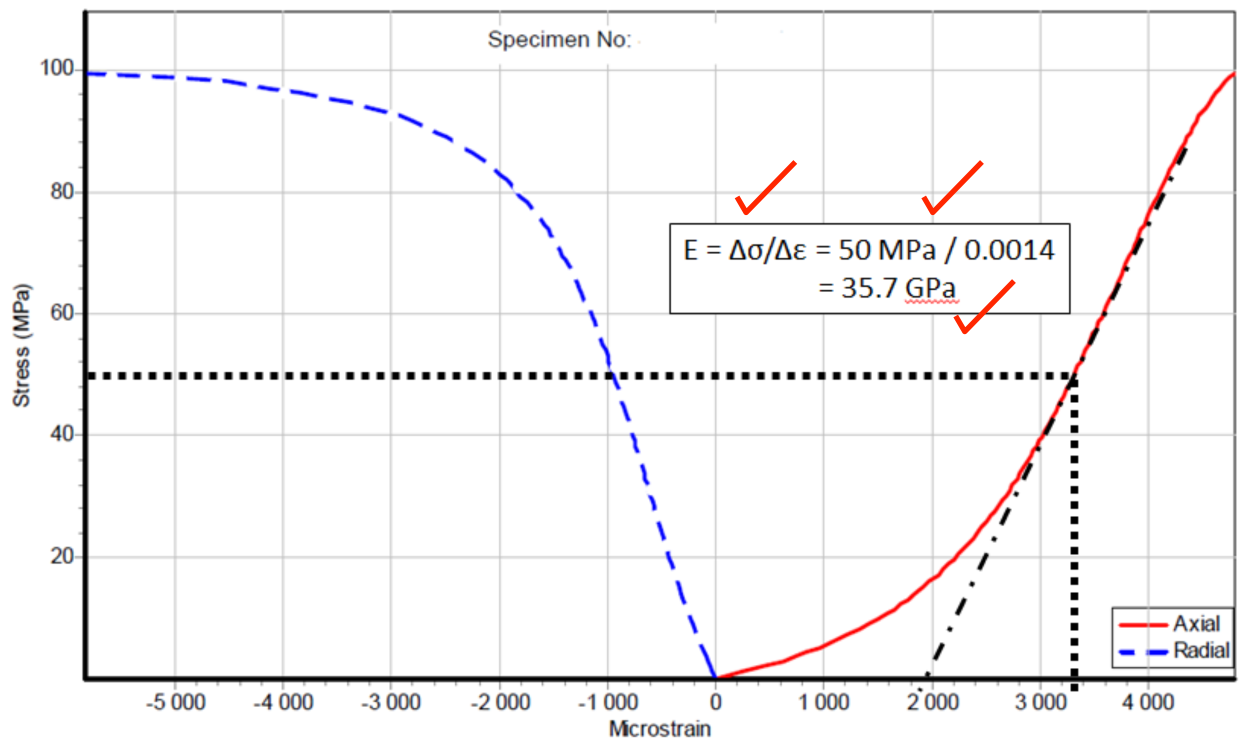
$\tau_{xy} = \tau_{yx}$ ✓ $\tau_{xz} = \tau_{zx}$ ✓ $\tau_{yz} = \tau_{zy}$ ✓

1.2 Given the stress-strain curve of uniaxial compressive testing below:

(a) Calculate the tangent elastic modulus at 50% strength. (3)

Assume peak strength of 100 MPa.

E = σ/ε = 35.7 MPa (accept 30-40 MPa)



(b) Describe the constitutive behaviour exhibited between 0 MPa and 40 MPa. (2)

Non-linear elastic.

(c) Describe and explain the behaviour exhibited between 70 MPa and 100 MPa.(3)

Onset of plastic strain and ductile deformation due to irreversible changes induced in the rock. Microcracks develop and coalesce to grow fractures, until a macroscopic fracture plane develops.

1.3 Given the Mohr's strain circle below. List the annotations (a) to (j). (5)

(a) $\gamma/2$

(b) ε

(c) ε_2

(d) ε_1

(e) ε_y

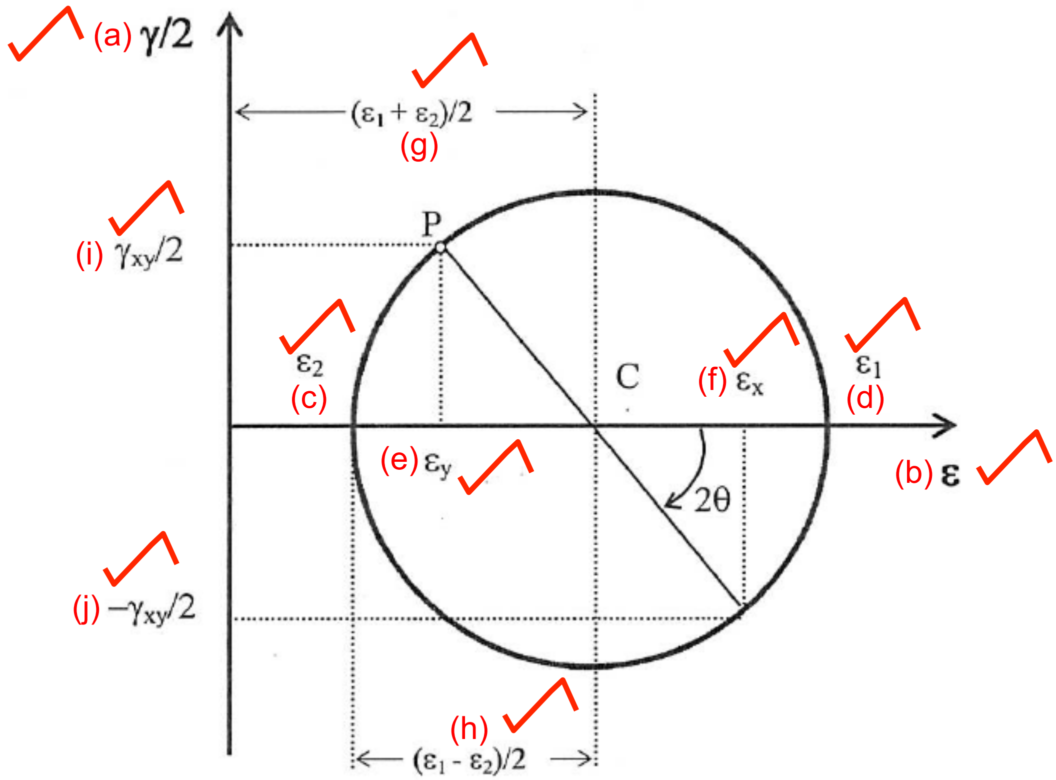
(f) ε_x

(g) $(\varepsilon_1 + \varepsilon_2)/2$

(h) $(\varepsilon_1 - \varepsilon_2)/2$

(i) $\gamma_{xy}/2$ or Γ_{xy} or ε_{xy}

(j) $-\gamma_{xy}/2$ or $-\Gamma_{xy}$ or $-\varepsilon_{xy}$



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QUESTION 2

A rock mass is transected by 3 intersecting joint sets. J_1 has 30 cm joint mean spacing, is continuous, with moderately rough unweathered surfaces, no infill and is dry. J_2 has a spacing of 20 cm, is planar and discontinuous, with signs of water and is moderately rough with some iron staining. J_3 is undulating and continuous, spaced 20-30 cm apart, relatively smooth, no infill and dry, but with iron staining. Intact rock uniaxial compressive strength is 80 MPa, and overburden density is 2720 kg/m^3.

2.1 Calculate the vertical virgin stress at a depth of 1500 m below surface. Comment on the ratio of rock strength to stress and likely consequences thereof. (3)

q_v = ρgh = 40 MPa UCS / q_v = 2 Stress = 50% UCS- likely to result in Stage II damage

2.2 Calculate number of joints per unit volume of the rock mass. (2)

Jv = 1/0.3 + 1/0.2 + 1/0.25 = 12 (also accept 13)

2.3 Estimate the RQD of the rock mass. (2)

RQD = 115 – (3.3 x 12) = 75.4 (accept 70 - 77)

2.4 List the six variables in Barton's Q. Determine their values and calculate the Q-rating of the rock mass using the attached tables. (7)

RQD = 75 (70 / 75 / 80) **Jr = 2 (1.5 - 3)** **Jw = 1**
Jn = 9 **Ja = 1 (0.75 - 2)** **SRF = 15 (10 - 20)**
Q = 75/9 x 2/1 x 1/15 = 1.1 (accept 0.3 - 3.6) Poor Rock

2.5 The Geomechanics Classification of the rock mass is 69, indicating Good Rock. Compare this to your Q-rating above and provide possible reasons for any difference between the ratings. (3)

RMR(Q) = 9 ln(1.1) + 44 = 45 (accept 33 – 55)
RMR does not take stress into account, hence overestimates this rock mass condition.

2.6 What adjustments are required to the basic RMR value to obtain an overall rock mass description? (2)

Adjusted RMR - Type of excavation (tunnels, foundations or slopes) and orientation of joints relative to the excavations (Very unfavourable, Unfavourable, Fair, Favourable, Very Favourable).

2.7 Use Kirsch equations to calculate the k-ratio that will result in zero stress on the periphery, at mid-height of the sidewall, of a circular tunnel mined through this rock mass. (6)

R/r = 1 **Θ = 0°**
Anywhere around periphery $\sigma_{rr} = 0$ and $\tau_{r\theta} = 0$
$\sigma_{\theta\theta}$ = ½ q (1 + k) (1 + 1) + ½ q (1 - k) (1 + 3) cos 0° = 0 MPa
q (1 + k) + 2 q (1 - k) = 0
k = 3

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QUESTION 3

3.1 Define the following: (5)

i. Normal stress

The perpendicular component of a stress acting on a surface.

ii. Shear stress

The parallel component of a stress acting on a surface.

iii. Isotropic rock mass

Reacts identically to the same stress applied in different directions.

iv. Lithostatic stress conditions

The stresses in all directions are equal.

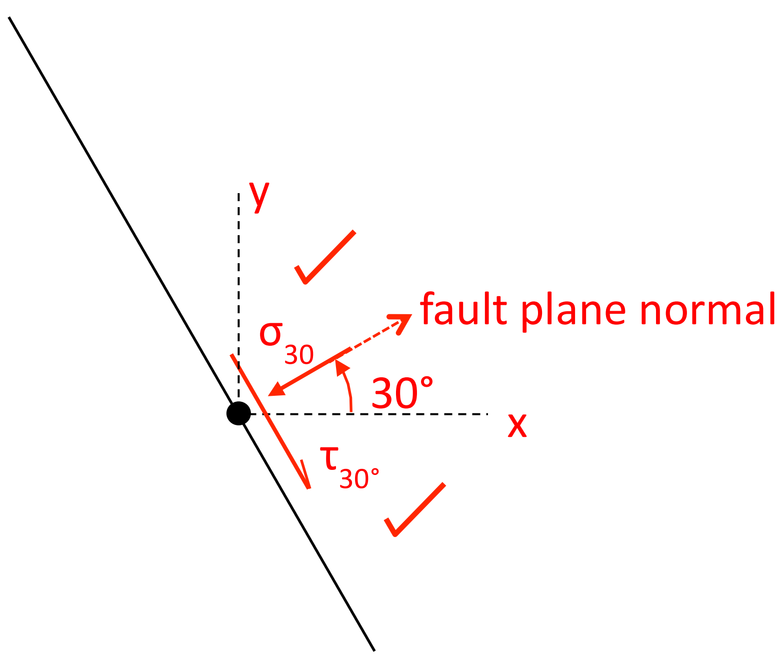
v. Principal stress

The maximum and minimum values of the normal stress components (shear stresses are zero).

3.2 The 2D virgin stress tensor as measured at a point of interest is given by:

$$\begin{pmatrix} \sigma_{xx} & \tau_{xy} \\ \tau_{yx} & \sigma_{yy} \end{pmatrix} = \begin{pmatrix} 40 & 8.66 \\ 8.66 & 50 \end{pmatrix} \text{ MPa}$$

Calculate and plot the normal and shear stresses acting on a 60° dipping fault as depicted below, when subjected to these virgin stresses. (5)



$\sigma_n = \sigma_{30°} = 40\cos^2(30°) + 17.32\sin(30°)\cos(30°) + 50\sin^2(30°)$
$= 30 + 7.5 + 12.5$
$= 50$ MPa

$\tau_{30°} = ½\,(50-40)\sin(60°) + 8.66\cos(60°)$
$= 4.33 + 4.33$
$= 8.66$ MPa

3.3 The normal and shear stress components at a point of interest on a fault plane are σ_n = 55 MPa and τ = 9.7 MPa. Calculate the minimum internal friction angle required for the fault to maintain stability, assuming no cohesion on the fault plane and Mohr-Coulomb failure. (3)

$\tau_n = c + \mu\sigma_n = 0 + 55\mu = 9.7$ MPa => **$\mu = 9.7 / 55 = 0.1764 = \tan(\varphi_i)$**

=> **$\varphi_i \geq 10°$**

3.4 Barton and Choubey developed the following equation to calculate shear strength on a plane:

$$\tau_p = \sigma_n \times \tan\left\{\varphi_b + JRC \times log_{10}\left({}^{JCS}/_{\sigma_n}\right)\right\}$$

(a) Describe a laboratory procedure to directly determine the base friction angle.(2)

Measure the tilt angle at which a smooth piece of core starts sliding over a second piece.

(b) For a joint roughness coefficient of 10 and joint wall compressive strength of 110 MPa, calculate the minimum base friction angle required to maintain stability at the point of interest for the fault described in Question 3.3 above. (3)

$\tau_p = 55\tan\{\varphi_b + 10\log_{10}(110/55)\} = 9.7$ MPa => **$\varphi_b \geq \arctan(9.7 / 55) - 10\log_{10}(2)$**

$\varphi_b \geq 6.99°$

(c) What significance is there with an increase in normal stress? (2)

Increased normal stress increases peak shear strength of discontinuities. Under extreme confinement, discontinuity shear strength approaches 'intact' rock shear strength.

3.5 Given an internal friction angle of 36° and base friction angle of 33°, calculate the normal stress at which the Barton-Choubey discontinuity shear strength is equal to the 'intact' rock Mohr-Coulomb shear strength on the fault plane described in Question 3.3 above. (5)

$\tau_n = c + \mu\sigma_n = \tau_p = \sigma_n \tan\{\varphi_b + JRC \log(JCS/\sigma_n)\}$

$\tan(\varphi_i) = \tan\{\varphi_b + JRC \log(JCS/\sigma_n)\}$

$\varphi_i - \varphi_b = JRC \log(JCS/\sigma_n)$

$$\sigma_n = \frac{JCS}{10^{\frac{\varphi_i - \varphi_b}{JRC}}} = \frac{110}{10^{\frac{36-33}{10}}} = 55\ \mathrm{MPa}$$

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QUESTION 4

4.1 A force of 1000 kN acts on the end-surface of a piece of core with original diameter 34 mm and length 71 mm. The core sample decreases in length to 65 mm and increases in diameter to 35 mm.

(a) Calculate the stress acting on the sample. (2)

r = 17 mm = 0.017 m **$A = \pi r^2 = 0.000908\ m^2$** **$\sigma = F/A = 1101$ MPa (= 1.10 GPa)**

(b) Calculate the axial and radial strain of the sample. (2)

$\varepsilon_a = \Delta\ell / L = 0.0845$ **$\varepsilon_r = \Delta d / D = -0.0294$**

(c) Calculate the Young's modulus of the material. (1)

$E = \sigma/\varepsilon = 13.0$ GPa

(d) Calculate the Poisson's ratio of the material. (2)

$\varepsilon_r = -\nu\varepsilon_a$ **$\nu = -\varepsilon_r / \varepsilon_a = 0.348$**

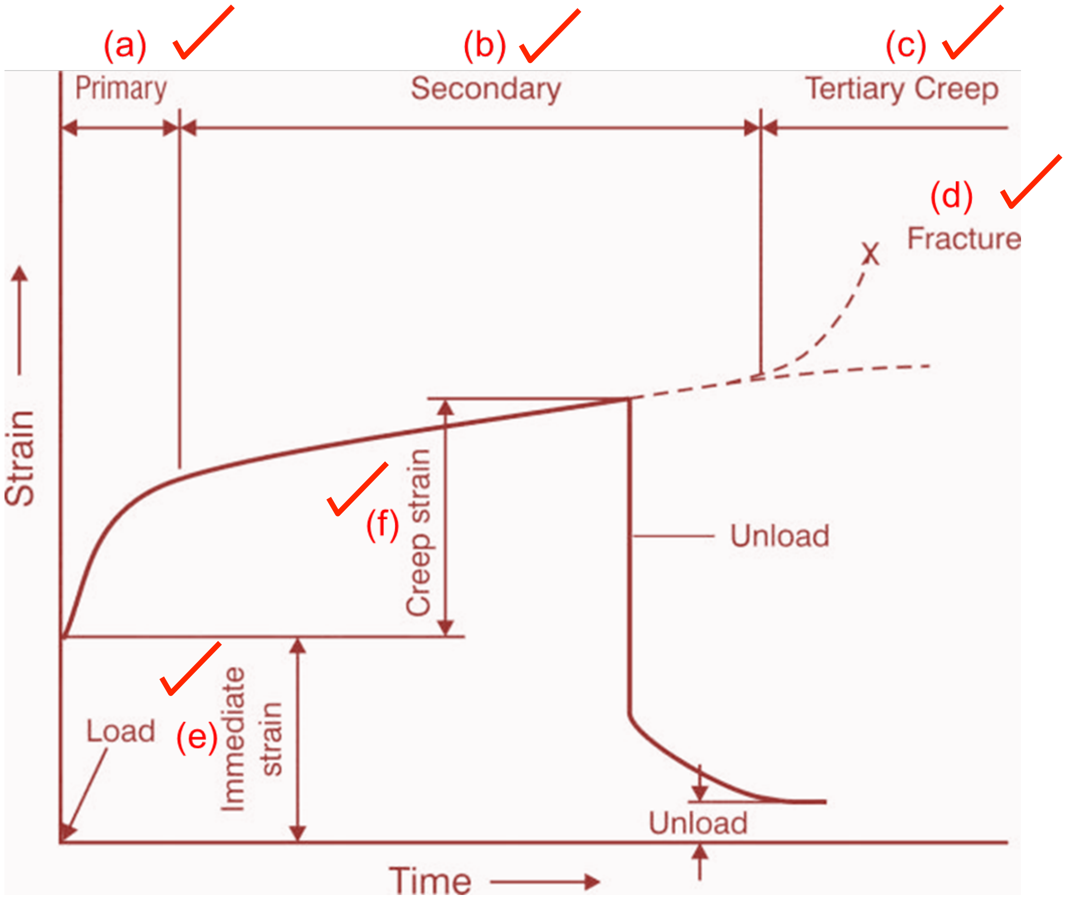
(e) Calculate the shear modulus of the material. (1)

G = 4.83 GPa

(f) Calculate the bulk modulus of the material. (1)

K = 14.3 GPa

4.2 The figure below illustrates a typical laboratory creep curve for rock suddenly subjected to a fixed elevated axial stress. List the annotations (a) to (f). (6)



4.3 Given the 3D strain tensor below (where Γ = γ/2), a Poisson's ratio of 0.25 and elastic modulus of 60 GPa.

$$\begin{pmatrix} \varepsilon_{xx} & \Gamma_{xy} & \Gamma_{xz} \\ \Gamma_{yx} & \varepsilon_{yy} & \Gamma_{yz} \\ \Gamma_{zx} & \Gamma_{zy} & \varepsilon_{zz} \end{pmatrix} = \begin{pmatrix} 0.0024 & 0.0012 & 0.0019 \\ 0.0012 & 0.0018 & 0.0003 \\ 0.0019 & 0.0003 & 0.0012 \end{pmatrix}$$

(a) Calculate the normal stress components, using generalized Hooke's Law equations. (6)

$$\boldsymbol{\sigma_{xx}} = \left[\frac{60\times10^9}{(1+0.25)(1-0.5)}\right][(1-0.25)2.4\times10^{-3} + 0.25(1.8\times10^{-3} + 1.2\times10^{-3})]\ \textbf{= 245 MPa}$$

$$\boldsymbol{\sigma_{yy}} = \left[\frac{60\times10^9}{(1+0.25)(1-0.5)}\right][(1-0.25)1.8\times10^{-3} + 0.25(2.4\times10^{-3} + 1.2\times10^{-3})]\ \textbf{= 216 MPa}$$

$$\boldsymbol{\sigma_{zz}} = \left[\frac{60\times10^9}{(1+0.25)(1-0.5)}\right][(1-0.25)1.2\times10^{-3} + 0.25(2.4\times10^{-3} + 1.8\times10^{-3})]\ \textbf{= 187 MPa}$$

(b) Explain why the maximum value of the Poisson's ratio for isotropic, linear elastic material such as rock at small strains is $\nu < 0.5$. (4)

Rock under compression undergoes volume decrease if it remains elastic (no failure). For that condition to be satisfied, Poisson's ratio should remain below 0.5 as any value greater than 0.5 implies volume increase that is failure.

Most materials have Poisson's ratio values ranging between 0.0 and 0.5. A perfectly incompressible material deformed elastically at small strains would have a Poisson's ratio of exactly 0.5. Rubber has a Poisson ratio of nearly 0.5 while cork is close to 0, showing very little lateral expansion when compressed. Most steels and rigid polymers when used within their design limits (before yield) exhibit values of about 0.3, increasing to 0.5 for post-yield deformation which occurs largely at constant volume.

Under conditions of simple tension and simple shear, all known materials experience displacements in the direction of force and increase in volume in case of hydrostatic tensile loading.

Some materials (known as auxetic materials) display a negative Poisson's ratio. When subjected to positive strain in a longitudinal axis, the transverse strain in the material will actually be positive (i.e. it would increase the cross sectional area).

The Poisson's ratio of a stable, isotropic, linear elastic materials will be greater than −1.0 or less than 0.5 because of the requirement for Young's modulus, the shear modulus and bulk modulus to have positive values.

An easy way of explaining this is to use the relationships between E, G, K and ν.

$$E = 2G(1+\nu) = 3K(1-2\nu)$$

As E, G and K cannot be negative;

$$(1+\nu) \geq 0 \quad \Rightarrow \quad \nu \geq -1$$

and

$$(1-2\nu) \geq 0 \quad \Rightarrow \quad \nu \leq 0.5$$

Combining the above: $-1 \leq \nu \leq 0.5$

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TOTAL MARKS: [100]