MEMORANDUM

CHAMBER OF MINES OF SOUTH AFRICA

MEMORANDUM

SUBJECT: ROCK MECHANICS CERTIFICATE PART 1: THEORY	EXAMINER: J WALLS
SUBJECT CODE:COMRMC1	MODERATOR: H YILMAZ
EXAMINATION DATE: OCTOBER 2015	TOTAL MARKS: 100
TIME: TBA	PASS MARK: 60

NUMBER OF MEMO PAGES:20 (*including Cover*)

THIS IS NOT AN OPENBOOK EXAMINATION – ONLY REFERENCES PROVIDED ARE ALLOWED

SPECIAL REQUIREMENTS:

1. Answer **all questions**. Answer the questions **legibly** in English.
2. Write your **ID Number** on the outside cover of each book used and on any graph paper or other loose sheets handed in.

   NB: Your name **must not** appear on any answer book or loose sheets.
3. Show all calculations **and check calculations on which the answers are based.**
4. Hand-held electronic calculators may be used for calculations. Reference notes may not be programmed into calculators.
5. Write **legibly** in ink on the **right hand page** only – **left hand pages will not be marked**.
6. Illustrate your answers by means of sketches or diagrams wherever possible.
7. **Final answers** must be given to an accuracy which is typical of practical conditions. However be careful not to use too few decimal places during your calculations, as rounding errors may result in incorrect answers.

   NB: Ensure that the correct units of measure (SI unit) are recorded as marks will be deducted from answers if the incorrect unit is used (even if the calculated value is correct).
8. In answering the questions, full advantage should be taken of your practical experience as well as data given.
9. Please note that you are not allowed to contact your examiner or moderator regarding this examination.
10. Cell phones are **NOT** allowed in the examination room.

MEMORANDUM

QUESTION 1 – MULTIPLE CHOICE (25)

An answer sheet has been provided at the end of the question paper. Complete the question using the answer sheet and submit together with the answer booklet. Each question represents 1 mark (correct) or 0 marks (incorrect). There is only one correct answer for each question.

ANSWER

No.	Question	Answer	
1.1	Express 300 000 N in kN	A	
		B	
		C	
		D	3×10^2 kN
1.2	In which area of the stress-strain curve is Young's modulus defined?	A	
		B	
		C	elastic deformation
		D	
1.3	In which Cartesian quadrants is the cosine trigonometric function positive?	A	
		B	
		C	
		D	first and fourth
1.4	The Hoek-Brown failure criterion is best applied to one of the following conditions:	A	
		B	Brittle failure of intact rock
		C	
		D	
1.5	What is measured by the ultrasonic waves method?	A	
		B	Sound wave velocity
		C	
		D	
1.6	Which of the following does not influence joint strength?	A	Joint spacing
		B	
		C	
		D	
1.7	RQD = 78; Jn = 6; Jr = 1.5; Ja = 2.0; Jw = 1.0; SRF = 2.5. What is the Q-value?	A	
		B	
		C	3.9
		D	

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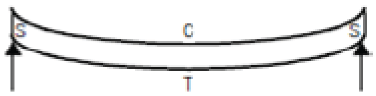
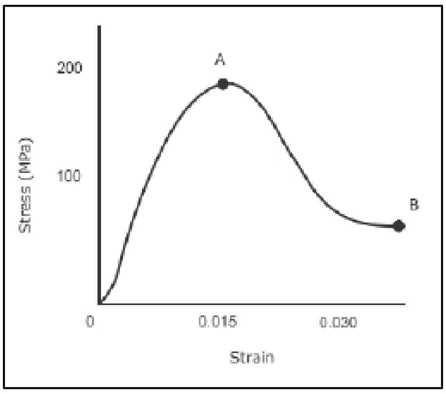
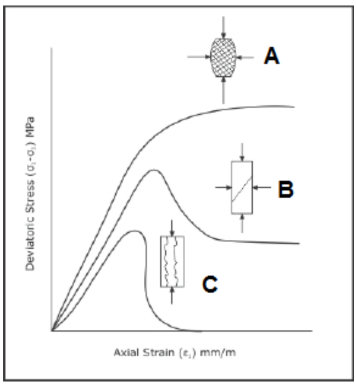
No.	Question	Answer	
1.8	Name the following beam: 	**A**	Freely supported beam
		B	
		C	
		D	
1.9	Determine the magnitude of the principal stresses for the given stress matrix with x-axis horizontal and y-axis vertical: $\begin{bmatrix} 35 & 15 \\ 15 & 22 \end{bmatrix}$	**A**	
		B	σ_1 = 44.84 MPa; σ_2 = 12.15 MPa
		C	
		D	
1.10	A negative shear stress . . .	**A**	
		B	
		C	
		D	Is plotted below the normal stress axis in the Mohr circle
1.11	Rocks that experience large amounts of plastic deformation before rupture can be classified as . . .	**A**	
		B	
		C	Ductile
		D	
1.12	Which failure model is presented by $\tau^2 = 4T_o(\sigma_n + T_o)$	**A**	
		B	
		C	
		D	Griffith
1.13	At 1 200 m below surface, which theory would result in the highest horizontal stress?	**A**	
		B	Heim's rule
		C	
		D	
1.14	Around a circular excavation, where the major applied stress is vertical and no internal pressure is applied, the normal stress σ_{rr} at the periphery of the excavation is equal to . . .	**A**	zero
		B	
		C	
		D	
1.15	In the Mohr-Coulomb failure relationship, the coefficient of friction is represented by . . .	**A**	μ
		B	
		C	
		D	
1.16	In the Mohr-Coulomb failure relationship, how does an increase in joint water pressure affect the shear strength of a failure surface?	**A**	
		B	
		C	
		D	All of the above

MEMORANDUM

No.	Question	Answer	
1.17	Using the Hoek-Brown relationship, calculate the peak strength of a sample if m = 15, s = 1.0, UCS = 200 MPa and confinement is 20 MPa.	**A**	
		B	
		C	
		D	336 MPa
1.18	Plastic deformation . . .	**A**	
		B	Is permanent
		C	
		D	
1.19	What behaviour is represented by the segment AB in the test graph? 	**A**	Strain softening
		B	
		C	
		D	
1.20	A "plane strain" analysis can be related to which of the following set of conditions . . .	**A**	
		B	
		C	$\sigma_{xx} \neq 0$, $\sigma_{yy} \neq 0$, $\sigma_{zz} \neq 0$, $\varepsilon_{zz} = 0$
		D	
1.21	Uni-axial loading can be described as . . .	**A**	$\sigma_1 \neq 0$, $\sigma_2 = 0$, $\sigma_3 = 0$
		B	
		C	
		D	
1.22	Provide the correct set of annotations for the stress strain curves at increasing confinement 	**A**	
		B	
		C	A = multiple shear fractures; B = shear fracture; C = axial splitting
		D	

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No.	Question		Answer
1.23	Define Poisson's ratio	A	
		B	
		C	
		D	None of the above
1.24	In Barton's equation to estimate the peak strength of joints, an increase in normal stress applied to the discontinuity surface, results in . . .	A	
		B	
		C	A combined increasing and decreasing effect on joint shear strength
		D	
1.25	Which of the following GSI ranges could refer to blocky/disturbed/seamy rock that has altered discontinuity surface quality?	A	
		B	30-40
		C	
		D	

TOTAL MARKS: [100]

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QUESTION 2 – STRESS AND STRAIN (20)

2.1 0.3 m long gauge lengths on a flat rock surface were used to measure deformation. The following measurements were recorded:

0.3003 m along the datum line (x-axis),

0.2997 m along the line at 60° from the datum line, and

0.3006 m along the line at 120° from the datum line.

2.1.1 Determine the magnitude and direction of the principal strains. [8]

2.1.2 Using the results in 2.1.1, determine the principal stresses, given E = 60 GPa and v = 0.25. [2]

Assume a horizontal x-axis (+ to the right) and a vertical y-axis (+ upwards). The z-axis increases into the rock mass and is orientated along the borehole axis. Data has been corrected for borehole end-effect.

2.2 Illustrate the strain gauge configuration in 2.1 and comment on the principal of measuring strain using an electrical resistance gauge (do not discuss the overcoring method). [3]

2.3 Define strain energy with the correct SI unit. [2]

2.4 Using the results in 2.1, calculate the strain energy density stored in the rock mass at this point. [3]

BONUS QUESTION

2.5 In-situ stress measurements were taken around a geological feature which was associated with seismic activity. Use the given 3D state of stress and directional cosines to determine the rotated normal stress components in all three directions. [6]

$$\text{Stress matrix} \begin{bmatrix} 12 & 1 & 2 \\ 1 & 8 & -1 \\ 2 & -1 & 5 \end{bmatrix} MPa$$

$$\text{Directional cosine} \begin{bmatrix} 0.9603 & 0.1571 & 0.2307 \\ -0.0740 & 0.9403 & -0.3323 \\ -0.2691 & 0.3020 & 0.9145 \end{bmatrix}$$

$$\sigma'_{xx} = \sigma_{xx}\lambda_{xx}^2 + \sigma_{yy}\lambda_{xy}^2 + \sigma_{zz}\lambda_{xz}^2 + 2\tau_{xy}\lambda_{xx}\lambda_{xy} + 2\tau_{yz}\lambda_{xy}\lambda_{xz} + 2\tau_{zx}\lambda_{xz}\lambda_{xx}$$

$$\sigma'_{yy} = \sigma_{xx}\lambda_{yx}^2 + \sigma_{yy}\lambda_{yy}^2 + \sigma_{zz}\lambda_{yz}^2 + 2\tau_{xy}\lambda_{yx}\lambda_{yy} + 2\tau_{yz}\lambda_{yy}\lambda_{yz} + 2\tau_{zx}\lambda_{yz}\lambda_{yx}$$

$$\sigma'_{zz} = \sigma_{xx}\lambda_{zx}^2 + \sigma_{yy}\lambda_{zy}^2 + \sigma_{zz}\lambda_{zz}^2 + 2\tau_{xy}\lambda_{zx}\lambda_{zy} + 2\tau_{yz}\lambda_{zy}\lambda_{zz} + 2\tau_{zx}\lambda_{zz}\lambda_{zx}$$

ANSWER

2.1.1 $\varepsilon_0 = -0.10\text{x}10^{-3}$; $\varepsilon_{60} = 0.10\text{x}10^{-3}$, $\varepsilon_{120} = -0.20\text{x}10^{-3}$ (3)

$\varepsilon_1 = 1.10\text{x}10^{-3}$, $\theta_1 = 50.45°$, $\varepsilon_2 = -2.43\text{x}10^{-3}$, $\theta_2 = 140.45°$ (5)

$\varepsilon_1 =$	**1.10**	**x10^{-3}**	$\theta_1 =$	**50.45**	**° from X**
$\varepsilon_2 =$	**-2.43**	**x10^{-3}**	$\theta_2 =$	**140.45**	**° from X**

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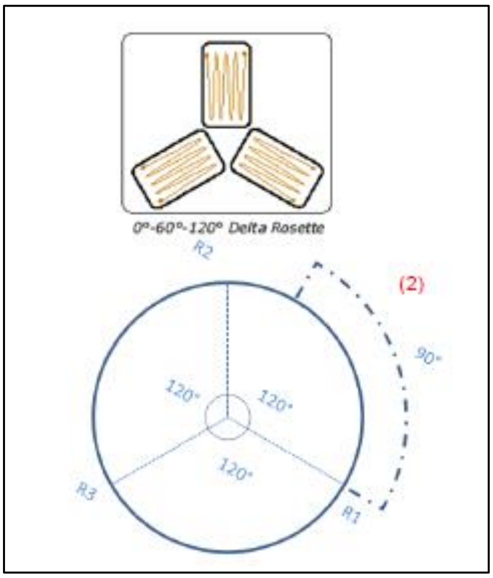
2.1.2 σ_1 = 31.33 MPa, σ_2 = -138.00 MPa (plane stress condition) (2)

σ_1 = 31.52 MPa

σ_2 = -137.92 MPa

2.2



The principal of strain measurement is based on the relationship between resistance of a conductive wire and strain. Increase in strain (elongation) of the wire is related to an increase in the resistance of the wire and vice versa*.* *[3]*

2.3 *Strain energy is the work done in deforming a body, measured in Joules (kN.m).* *[2]*

2.4 $w = \frac{1}{2}(\sigma_1\varepsilon_1 + \sigma_2\varepsilon_2 + \sigma_3\varepsilon_3)$ *= 0.185 MJ.m^{-3}* *[3]*

BONUS QUESTION

2.5 *σ_{xx}' = 12.6 MPa*

σ_{yy}' = 8.3 MPa

σ_{zz}' = 4.1 MPa. *[6]*

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QUESTION 3 – ROCK STRENGTH (15)

Quartzite specimens are tested for triaxial compressive strength and the following Mohr-Coulomb equation is derived from the test results:

$$\tau = 25 + \sigma_n \tan 40^o$$

3.1. Plot the Mohr-Coulomb failure line *[4]*

3.2. Determine graphically if the following stress states would cause failure:

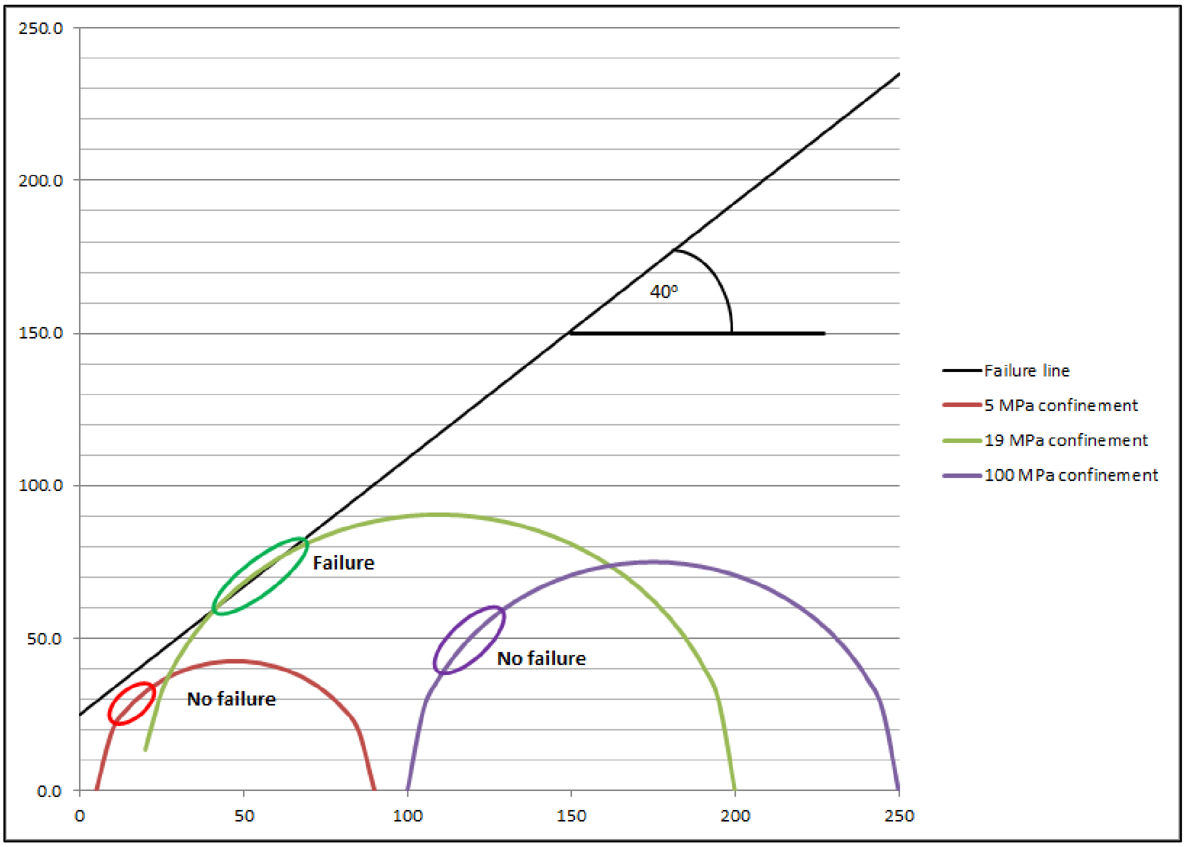
Confinement (MPa)	***Major stress (MPa)***
5	*90*
19	*200*
100	*250*

[9]

3.3. Explain the reason for your decision on failure in (4.2.) *[2]*

ANSWER

3.1. and 3.2.



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3.3.

Confinement (MPa)	***Major stress (MPa)***	***Failure***	***Reason***
5	*90*	*No*	*Mohr circle does not intersect the failure line*
19	*200*	*Yes*	*Mohr circle intersects the failure line*
100	*250*	*No*	*Mohr circle does not intersect the failure line*

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QUESTION 4 – STRESS IN ROCK AND ROCKMASSES (25)

A vertical tunnel (shaft), 6 m in diameter will be bored from surface to 1 200 m below surface. The k-ratio in the y-direction, k_y = 1.4 and in the x-direction, k_x = 0.8 at the maximum depth. The average host rock density is $2.9x10^3$ $kg.m^{-3}$, UCS = 130 MPa and the rock mass condition is generally good (Q = 15).

4.1 Discuss the parameters that comprise the Hoek-Brown failure criterion and name two limitations to this theory. [4]

4.2 Calculate the m and s parameter values using the relations:

$$m = 10\,x\,e^{\frac{(RMR-100)}{14}} \quad and \quad s = e^{\frac{(RMR-100)}{9}}$$

[2]

4.3 Calculate the radial and tangential stresses in the shaft sidewall in the x-direction, as shown in the table, using the Kirsch equations. Draw a sketch to illustrate the scenario. Present your results in a table.

Depth into the sidewall, x-direction (m)	σ_{rr}	$\sigma_{\theta\theta}$
0.00		
0.50		
1.50		
2.00		

4.4 Using the parameters and results obtained in 4.14.2 and 4.3, estimate the depth of failure that might occur in the side-wall, based on the Hoek-Brown failure criterion (assume that the radial and tangential stresses are also principal stresses). Present your results in a table.

ANSWER

4.1 $\sigma_1 = \sigma_3 + \sqrt{m\sigma_c\sigma_3 + s\sigma_c^2}$

$\sigma_1 = major\ principal\ stress$

$\sigma_3 = minor\ principal\ stress$

$\sigma_c = UCS$

m, s = rock property constants

m = f(rock type)

s = f(fracturing / weathering)

limitations (2 limitations required): suitable for brittle failure, less accurately representative of jointed rock masses, isotropic conditions, overstate downgrading ("m", "s"), over-estimate high confinements *[4]*

4.2 *m = 1.04; s = 0.03* *[2]*

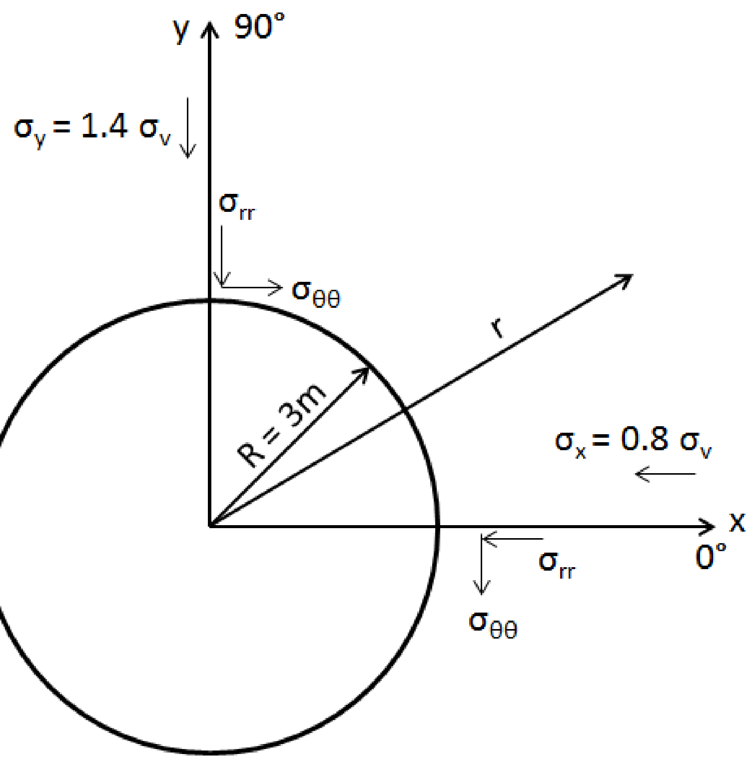
4.3

σ_y	σ_x	"k" = σ_x / σ_y
"q"	"σ_h"	
47.8	27.3	0.57

(3)

Depth into the sidewall, x-direction (m)	*σ_{rr} (MPa)*	*$\sigma_{\theta\theta}$ (MPa)*
0.00	*0.0*	*116.1*
0.50	*13.2*	*92.0*
1.50	*22.8*	*70.6*
2.00	*24.6*	*65.3*

(8)



(3)

4.4

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Depth into the sidewall (m)	σ_{rr} *(MPa)*	$\sigma_{\theta\theta}$ *(MPa)*	*HB strength (MPa)*	*Failure*
0.00	*0.0*	*116.1*	*13.8*	*Y*
0.50	*13.2*	*92.0*	*49.2*	*Y*
1.50	*22.8*	*70.6*	*68.5*	*Y*
2.00	*24.6*	*65.3*	*71.9*	*N*

[4]

4.5 2.0 m long tendons *[1]*

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QUESTION 5 – ROCK TESTING (15)

5.1 Explain Heim's law *[2]*

5.2 The virgin or primitive state of stress is a function of several influencing components. Discuss. *[4]*

5.3 Briefly outline in a table the principles and applications of the following three methods of stress measurement:

- *Strain cells*
- *Acoustic emission*
- *Jacking (flat jack)*

. *[9]*

ANSWER

5.1 All normal stress components are equal and depth independent at depth and where the rock stress tends to be lithostatic. Vertical and horizontal stresses are therefore principal, equal and a function of density, gravitational acceleration and depth below surface. Heim's law assumes that if a rockmass is left undisturbed for long enough shear stresses will disappear in time through a process of creep, making the normal stress components the principal stresses. [2]

5.2

- Weight of the overlying strata in terms of depth, average overburden density and gravitational acceleration;
- Superimposed tectonic stress, which is derived from regional forces that shape the crust and determine its structure
- Local stress variations imposed by individual geological structures such as faults, dykes, sill s and joints
- Residual stresses, which endure in the rock long after the forces that caused them have dissipated, and
- Pore water pressure

5.3 . [9]

MEMORANDUM

METHOD	PRINCIPLES	APPLICATION
Strain cells	Overcoring Doorstopper strain cell; CSIR strain cell; CSIRO Hi cell State of stress is determined from three normal strain components measured in a particular orientation installed in a borehole drilled along the edge of the excavation. The doorstopper is then overcored resulting in a stress/strain relief and strain gauges are used to measure the strain relief. The strains are converted into stresses using the host rock elastic constants appropriate to small gauge lengths. At least three orthogonal boreholes are necessary to determine the state of 3D field stress in the rock mass.	In situ measuring technique "Doorstopper" for stress measurement around excavations or geological structures.
Acoustic emission	Defined as "a naturally occurring phenomenon" whereby external stimuli, such as mechanical loading, generate sources of elastic waves. The method is based on the ability of in-situ rock to develop a memory of the stress field it was subjected to, in its confining environment. The Kaiser effect is a phenomenon associated with a significant increase in the acoustic emission rate when the new reapplied stress exceeds the original confining stress, in a rock that was originally confined and then relaxed.	Smaller cores are drilled from a large core that was removed from the stressed environment. The directions of the secondary cores relative to the original core are important. The smaller cores are subjected to uni-axial tests in a laboratory environment. Gives direct measure of stress and reduces experimental errors. Interference from mining is minimised or eliminated in the case of greenfield study sites.
Jacking (flat jack)	A direct, non-destructive method performed in-situ. Similar to an envelope with inlet and outlet ports pressurised with hydraulic oil. Non-destructive testing method, to determine: – Compressive stress – Deformability properties – Load applied to the sample.	For measurements of near-surface rock stresses in exposed roof, walls, or floor of excavations, on surface or underground.

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$$\sigma_{nn} = \sigma_{xx}\cos^2\theta + 2\tau_{xy}\sin\theta\cos\theta + \sigma_{yy}\sin^2\theta \qquad \sigma'_{xx} = \sigma_{xx}\cos^2\theta + 2\tau_{xy}\sin\theta\cos\theta + \sigma_{yy}\sin^2\theta$$

$$\sigma_{mm} = \sigma_{xx}\sin^2\theta - 2\tau_{xy}\sin\theta\cos\theta + \sigma_{yy}\cos^2\theta \qquad \sigma'_{yy} = \sigma_{xx}\sin^2\theta - 2\tau_{xy}\sin\theta\cos\theta + \sigma_{yy}\cos^2\theta$$

$$\tau_{nm} = \frac{1}{2}(\sigma_{yy} - \sigma_{xx})\sin 2\theta + \tau_{xy}\cos 2\theta \qquad \tau'_{xy} = \frac{1}{2}(\sigma_{yy} - \sigma_{xx})\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$\tan 2\theta = \frac{2\tau_{xy}}{\sigma_{xx} - \sigma_{yy}} \qquad \sigma_1 + \sigma_2 = \sigma_{xx} + \sigma_{yy}$$

$$\sigma_1 = \frac{1}{2}(\sigma_{xx} + \sigma_{yy}) + \frac{1}{2}\sqrt{(\sigma_{xx} - \sigma_{yy})^2 + 4\tau_{xy}^2} \qquad \sigma'_{xx} + \sigma'_{yy} = \sigma_{xx} + \sigma_{yy}$$

$$\sigma_2 = \frac{1}{2}(\sigma_{xx} + \sigma_{yy}) - \frac{1}{2}\sqrt{(\sigma_{xx} - \sigma_{yy})^2 + 4\tau_{xy}^2} \qquad \tau_{\max} = \frac{1}{2}\sqrt{(\sigma_{xx} - \sigma_{yy})^2 + 4\tau_{xy}^2}$$

$$\sigma_{nn} = \sigma_1\cos^2\theta + \sigma_2\sin^2\theta \qquad \tau_{\max} = \frac{1}{2}(\sigma_1 - \sigma_2)$$

$$\sigma_{mm} = \sigma_1\sin^2\theta + \sigma_2\cos^2\theta$$

$$\sigma_{nn} = \frac{1}{2}(\sigma_1 + \sigma_2) + \frac{1}{2}(\sigma_1 - \sigma_2)\cos 2\theta \qquad \tau_{nm} = -\frac{1}{2}(\sigma_1 - \sigma_2)\sin 2\theta$$

$$\sigma_{mm} = \frac{1}{2}(\sigma_1 + \sigma_2) - \frac{1}{2}(\sigma_1 - \sigma_2)\cos 2\theta$$

$$\gamma_{xy} = \gamma_{yx} = 2\varepsilon_{xy} = 2\varepsilon_{yx}, \qquad \gamma_{yz} = \gamma_{zy} = 2\varepsilon_{yz} = 2\varepsilon_{zy}, \qquad \gamma_{zx} = \gamma_{xz} = 2\varepsilon_{zx} = 2\varepsilon_{xz}$$

$$\varepsilon_n = \varepsilon_{xx}\cos^2\theta + \gamma_{xy}\sin\theta\cos\theta + \varepsilon_{yy}\sin^2\theta \qquad \varepsilon'_{xx} = \varepsilon_{xx}\cos^2\theta + \gamma_{xy}\sin\theta\cos\theta + \varepsilon_{yy}\sin^2\theta$$

$$\gamma_{nm} = (\varepsilon_{yy} - \varepsilon_{xx})\sin 2\theta + \gamma_{xy}\cos 2\theta \qquad \varepsilon'_{yy} = \varepsilon_{xx}\sin^2\theta - \gamma_{xy}\sin\theta\cos\theta + \varepsilon_{yy}\cos^2\theta$$

$$\tan 2\theta = \frac{\gamma_{xy}}{\varepsilon_{xx} - \varepsilon_{yy}} \qquad \varepsilon_1 + \varepsilon_2 = \varepsilon_{xx} + \varepsilon_{yy} \qquad \gamma'_{xy} = (\varepsilon_{yy} - \varepsilon_{xx})\sin 2\theta + \gamma_{xy}\cos 2\theta$$

$$\varepsilon_1 = \frac{1}{2}(\varepsilon_{xx} + \varepsilon_{yy}) + \frac{1}{2}\sqrt{(\varepsilon_{xx} - \varepsilon_{yy})^2 + \gamma_{xy}^2} \qquad \varepsilon'_{xx} + \varepsilon'_{yy} = \varepsilon_{xx} + \varepsilon_{yy}$$

$$\varepsilon_2 = \frac{1}{2}(\varepsilon_{xx} + \varepsilon_{yy}) - \frac{1}{2}\sqrt{(\varepsilon_{xx} - \varepsilon_{yy})^2 + \gamma_{xy}^2} \qquad \gamma_{\max} = \sqrt{(\varepsilon_{xx} - \varepsilon_{yy})^2 + \gamma_{xy}^2}$$

$$\varepsilon_n = \varepsilon_1\cos^2\theta + \varepsilon_2\sin^2\theta \qquad \gamma_{\max} = (\varepsilon_1 - \varepsilon_2)$$

$$\varepsilon_n = \frac{1}{2}(\varepsilon_1 + \varepsilon_2) + \frac{1}{2}(\varepsilon_1 - \varepsilon_2)\cos 2\theta \qquad \varepsilon = \frac{\delta\ell}{\ell_o}$$

$$\gamma_{nm} = -(\varepsilon_1 - \varepsilon_2)\sin 2\theta$$

$$\sigma_{rr} = \sigma_{xx}\cos^2\theta + 2\tau_{xy}\sin\theta\cos\theta + \sigma_{yy}\sin^2\theta \qquad \varepsilon_{rr} = \varepsilon_{xx}\cos^2\theta + \gamma_{xy}\sin\theta\cos\theta + \varepsilon_{yy}\sin^2\theta$$

$$\sigma_{\theta\theta} = \sigma_{xx}\sin^2\theta - 2\tau_{xy}\sin\theta\cos\theta + \sigma_{yy}\cos^2\theta \qquad \varepsilon_{\theta\theta} = \varepsilon_{xx}\sin^2\theta - \gamma_{xy}\sin\theta\cos\theta + \varepsilon_{yy}\cos^2\theta$$

$$\tau_{r\theta} = \frac{1}{2}(\sigma_{yy} - \sigma_{xx})\sin 2\theta + \tau_{xy}\cos 2\theta \qquad \gamma_{r\theta} = (\varepsilon_{yy} - \varepsilon_{xx})\sin 2\theta + \gamma_{xy}\cos 2\theta$$

$$w = \frac{1}{2}(\sigma_1\varepsilon_1 + \sigma_2\varepsilon_2 + \sigma_3\varepsilon_3) \qquad w = \frac{1}{2}(\sigma_{xx}\varepsilon_{xx} + \sigma_{yy}\varepsilon_{yy} + \sigma_{zz}\varepsilon_{zz} + \tau_{xy}\gamma_{xy} + \tau_{yz}\gamma_{yz} + \tau_{zx}\gamma_{zx})$$

$$Q = wV$$

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$$\varepsilon_{1,2} = \frac{(\varepsilon_0 + \varepsilon_{90})}{2} \pm \frac{\sqrt{[(\varepsilon_0^2 - \varepsilon_{90}^2) + (2\varepsilon_{45} + \varepsilon_0 - \varepsilon_{90})^2]}}{2}$$

$$\tan 2\theta = \frac{(2\varepsilon_{45} - \varepsilon_0 - \varepsilon_{90})}{(\varepsilon_0 - \varepsilon_{90})}$$

$$\varepsilon_{1,2} = \frac{(\varepsilon_0 + \varepsilon_{60} + \varepsilon_{120})}{3} \pm \left(\frac{2}{3}\right) \sqrt{[\varepsilon_0^2 + \varepsilon_{60}^2 + \varepsilon_{120}^2 - (\varepsilon_0\varepsilon_{60} + \varepsilon_{60}\varepsilon_{120} + \varepsilon_{120}\varepsilon_0)]}$$

$$\tan 2\theta = \left(\sqrt{3}\right)\frac{(\varepsilon_{60} - \varepsilon_{120})}{(2\varepsilon_0 - \varepsilon_{60} - \varepsilon_{120})}$$

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$$\varepsilon_{xx} = \frac{1}{E}\left\{\sigma_{xx} - \nu(\sigma_{yy} + \sigma_{zz})\right\} \quad \gamma_{xy} = \frac{1}{G}\tau_{xy} \quad \varepsilon_{xy} = \frac{1}{2G}\tau_{xy} \quad G = \frac{E}{2(1+\nu)}$$

$$\varepsilon_{yy} = \frac{1}{E}\left\{\sigma_{yy} - \nu(\sigma_{xx} + \sigma_{zz})\right\} \quad \gamma_{yz} = \frac{1}{G}\tau_{yz} \quad \varepsilon_{yz} = \frac{1}{2G}\tau_{yz} \quad K = \frac{E}{3(1 - 2\nu)}$$

$$\varepsilon_{zz} = \frac{1}{E}\left\{\sigma_{zz} - \nu(\sigma_{xx} + \sigma_{yy})\right\} \quad \gamma_{zx} = \frac{1}{G}\tau_{zx} \quad \varepsilon_{zx} = \frac{1}{2G}\tau_{zx}$$

$$\sigma_{xx} = \lambda\Delta + 2G\varepsilon_{xx} \quad \tau_{xy} = G\gamma_{xy} \quad \tau_{xy} = 2G\varepsilon_{xy} \quad \Delta = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}$$

$$\sigma_{yy} = \lambda\Delta + 2G\varepsilon_{yy} \quad \tau_{yz} = G\gamma_{yz} \quad \tau_{yz} = 2G\varepsilon_{yz} \quad \lambda = \frac{E\nu}{(1+\nu)(1-2\nu)}$$

$$\sigma_{zz} = \lambda\Delta + 2G\varepsilon_{zz} \quad \tau_{zx} = G\gamma_{zx} \quad \tau_{zx} = 2G\varepsilon_{zx}$$

$$\sigma_{zz} = 0 \quad \varepsilon_{zz} = -\frac{\nu}{E}(\sigma_{xx} + \sigma_{yy})$$

$$\varepsilon_{xx} = \frac{1}{E}(\sigma_{xx} - \nu\sigma_{yy}) \quad \gamma_{xy} = \frac{1}{G}\tau_{xy} \quad \sigma_{xx} = \frac{E}{1-\nu^2}(\varepsilon_{xx} + \nu\varepsilon_{yy}) \quad \tau_{xy} = G\gamma_{xy}$$

$$\varepsilon_{yy} = \frac{1}{E}(\sigma_{yy} - \nu\sigma_{xx}) \quad \sigma_{yy} = \frac{E}{1-\nu^2}(\varepsilon_{yy} + \nu\varepsilon_{xx})$$

$$\varepsilon_{zz} = 0 \quad \sigma_{zz} = \nu(\sigma_{xx} + \sigma_{yy})$$

$$\varepsilon_{xx} = \frac{1}{E}\left\{ (1-\nu^2)\sigma_{xx} - \nu(1+\nu)\sigma_{yy}\right\}$$

$$\varepsilon_{yy} = \frac{1}{E}\left\{ (1-\nu^2)\sigma_{yy} - \nu(1+\nu)\sigma_{xx}\right\}$$

$$q = \rho g H \quad \sigma_v = q \quad \sigma_h = kq$$

$$\sigma_{rr} = \frac{1}{2}q(1+k)\left(1 - \frac{R^2}{r^2}\right) - \frac{1}{2}q(1-k)\left(1 - \frac{4R^2}{r^2} + \frac{3R^4}{r^4}\right)\cos 2\theta$$

$$\sigma_{\theta\theta} = \frac{1}{2}q(1+k)\left(1 + \frac{R^2}{r^2}\right) + \frac{1}{2}q(1-k)\left(1 + \frac{3R^4}{r^4}\right)\cos 2\theta$$

$$\tau_{r\theta} = \frac{1}{2}q(1-k)\left(1 + \frac{2R^2}{r^2} - \frac{3R^4}{r^4}\right)\sin 2\theta$$

$$\sigma_{rr} = q\left(1 - \frac{R^2}{r^2}\right) \quad \sigma_{\theta\theta} = q\left(1 + \frac{R^2}{r^2}\right) \quad \tau_{r\theta} = 0 \quad \sigma_{zz} = 2\nu q$$

$$u = \frac{(1+\nu)}{E}q\left\{(1-2\nu)r + \frac{R^2}{r}\right\} \quad v = 0$$

MEMORANDUM

$$s_z = \frac{2(1-\nu)q}{G}\sqrt{\ell^2 - x^2} \qquad \ell_c = \frac{s_m G}{2(1-\nu)q}$$

$$\ell < \ell_c \quad ERR = \frac{\Delta U_m}{\Delta A} = \frac{\pi(1-\nu)\ell q^2}{2G} \qquad \ell = \ell_c \quad ERR = \frac{\Delta U_m}{\Delta A} = \frac{\pi}{4} s_m q$$

$$\ell > \ell_c \quad ERR = \frac{\Delta U_m}{\Delta A} = \frac{\pi}{4} s_m q\left(\frac{1-(1-\ell_c/\ell)^{1,27}}{\ell_c/\ell}\right) \qquad \ell \to \infty \quad ERR = \frac{\Delta U_m}{\Delta A} = s_m q$$

$$\tau = c + \mu\sigma \qquad \mu = \tan\phi \qquad \sigma_1 = \sigma_c + \tan\psi\,\sigma_3$$

$$\tan\psi = \frac{1+\sin\phi}{1-\sin\phi} \qquad \sigma_c = \frac{2c\cos\phi}{1-\sin\phi} \qquad \phi = \operatorname{asin}\left(\frac{\tan\psi - 1}{\tan\psi + 1}\right) \qquad c = \frac{\sigma_c}{2\sqrt{\tan\psi}}$$

$$Q = \frac{RQD}{J_n}\frac{J_r}{J_a}\frac{J_w}{SRF} \qquad RMR = 9\log_e Q + 44$$

$$\sigma_1 = \sigma_3 + \sigma_{ci}\left(m_b\frac{\sigma_3}{\sigma_{ci}} + s\right)^a \qquad \sigma_1 = \sigma_3 + \sigma_{ci}\sqrt{m_i\frac{\sigma_3}{\sigma_{ci}} + 1}$$

$$GSI > 25 \quad GSI = RMR_{76} \qquad GSI = RMR_{89} - 5$$

$$GSI < 25 \quad Q' = \frac{\mathrm{RQD}}{\mathrm{J_n}}\frac{J_r}{J_a} \qquad GSI = 9\log_e Q' + 44$$

$$m_b = m_i \exp\left(\frac{GSI - 100}{28}\right)$$

$$GSI > 25 \quad s = \exp\left(\frac{GSI-100}{9}\right) \qquad a = \frac{1}{2} \qquad \sigma_1 = \sigma_3 + \sigma_{ci}\sqrt{m_b\frac{\sigma_3}{\sigma_{ci}} + s}$$

$$\sigma_{tm} = \frac{\sigma_{ci}}{2}\left(m_b - \sqrt{m_b^2 + 4s}\right) \qquad \sigma_{cm} = \sigma_{ci}\sqrt{s}$$

$$GSI < 25 \quad s = 0 \qquad a = 0.65 - \frac{GSI}{200} \qquad \sigma_1 = \sigma_3 + \sigma_{ci}\left(m_b\frac{\sigma_3}{\sigma_{ci}}\right)^{0,65-\frac{GSI}{200}}$$

$$\sigma_{tm} = 0 \qquad \sigma_{cm} = 0$$

$$\sigma_{ci} > 100\ \mathrm{MPa} \qquad \mathrm{E_m(GPa)} = 10^{\left(\frac{\mathrm{GSI-10}}{40}\right)}$$

$$\sigma_{ci} < 100\ \mathrm{MPa} \qquad \mathrm{E_m(GPa)} = \sqrt{\frac{\sigma_{\mathrm{ci}}(MPa)}{100(MPa)}}\,10^{\left(\frac{\mathrm{GSI-10}}{40}\right)}$$

$$\tau = \sigma_n \tan\left[\phi_a + JRC\log_{10}\left(\frac{JCS}{\sigma_n}\right)\right]$$

$$\tau_{strength} = \sigma_n \tan\left[JRC\log_{10}\left(\frac{JCS}{\sigma_n}\right) + \varphi_\gamma\right]$$

MEMORANDUM

$$q = 0.025H \quad MPa \quad H[m] \qquad \sigma_v = \rho g h \qquad \sigma_h = k\sigma_v$$

$$e = 1 - \frac{w^2}{C^2} \qquad C = w + B \qquad r = \frac{h}{M}\left(1 - \frac{w^2}{C^2}\right) \qquad h_e = h\left(1 - \frac{w^2}{C^2}\right) \qquad h_e = he$$

$$\sigma_p = \frac{q}{1-e} \qquad \sigma_p = 0.025H\,\frac{(w+B)^2}{w^2} \quad MPa \qquad R = \frac{w}{h}$$

$R < 5$

$$\sigma_s = 7.2\frac{w^{0.46}}{h^{0.66}} \quad MPa \qquad Fs = \frac{\sigma_s}{\sigma_p} \qquad Fs = 288\frac{w^{2.46}}{Hh^{0.66}(w+B)^2}$$

$R \geq 5$

$$\sigma_s = 7.2\frac{R_o^{0.5933}}{V^{0.0667}}\left\{\frac{0.5933}{\varepsilon}\left[\left(\frac{R}{R_o}\right)^{\varepsilon} - 1\right] + 1\right\} \quad MPa$$

$$V = w^2 h \qquad R_o = 5 \qquad \varepsilon = 2.5$$

$$\sigma_s = \frac{0.07942}{(w^2 h)^{0.0667}}\left(R^{2.5} + 179.65\right) \quad MPa$$

$$Fs = \frac{\sigma_s}{\sigma_p} \qquad Fs = \frac{3.1768w^{1.8666}}{H(w+B)^2 h^{0.0667}}\left\{\left(\frac{w}{h}\right)^{2.5} + 179.65\right\}$$

$$e = 1 - \frac{w_1 w_2}{C_1 C_2} \qquad w_e = \frac{4w_1 w_2}{2w_1 + 2w_2}$$

$$\sigma_p = 0.025H\,\frac{C_1 C_2}{w_1 w_2} \quad MPa$$

$$R = \frac{E}{4(1-\nu^2)}\,\frac{s_m}{(\sigma_{cr} - q)} \qquad R = 0.28\frac{s_m}{\varepsilon_{cr}}$$

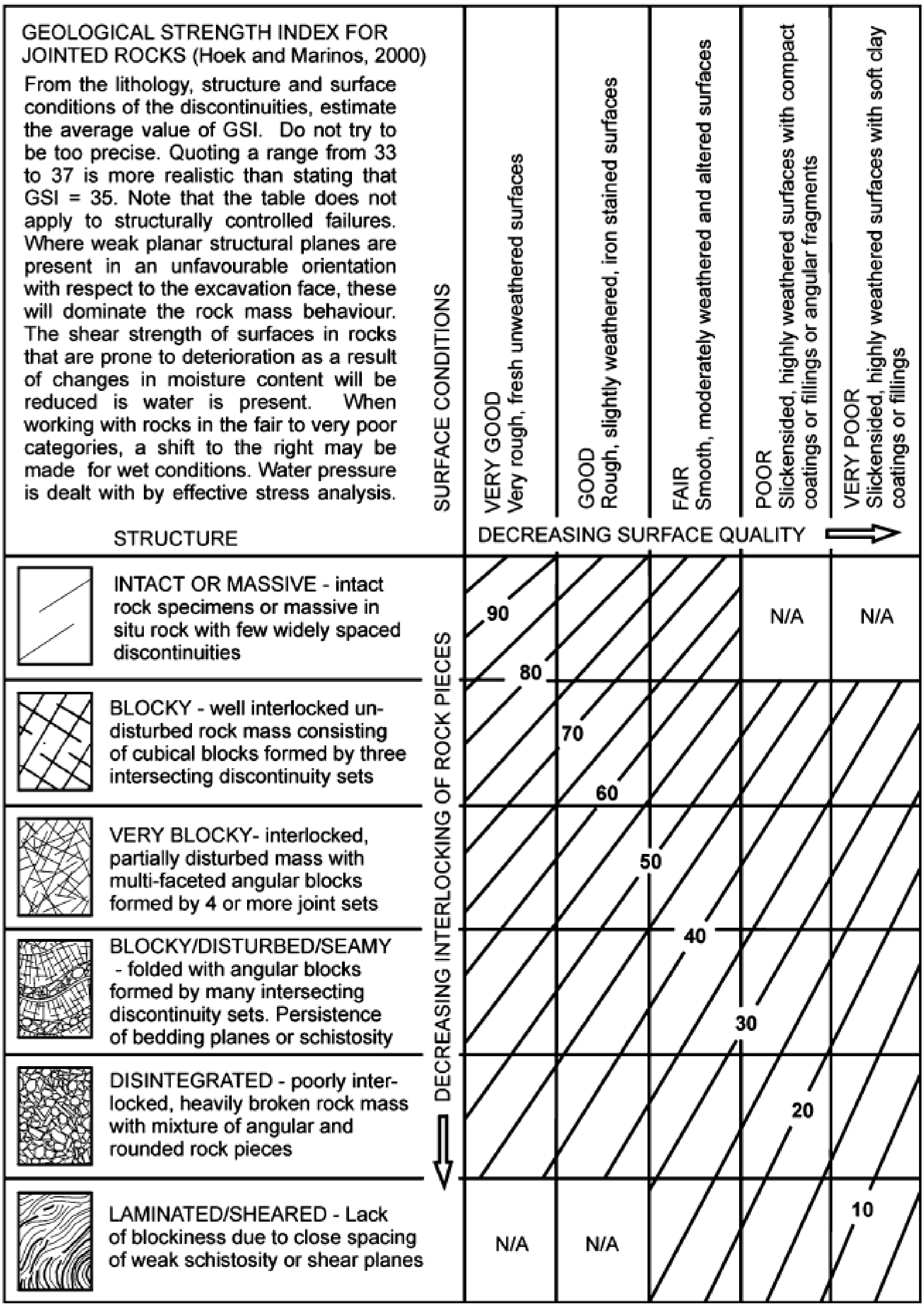
$$v_p = \sqrt{\frac{\lambda + 2G}{\rho}} \qquad v_s = \sqrt{\frac{G}{\rho}}$$

$$ESS = |\tau| - [\sigma_n \tan(\theta) + C_0]$$

$$\sigma_1 = \sigma_3 + \sqrt{m\sigma_c\sigma_3 + s\sigma_c^2}$$

$$\tau_{strength} = \sigma_n \tan\left[JRC \log_{10}\left(\frac{JCS}{\sigma_n}\right) + \varphi_\gamma\right]$$

MEMORANDUM


GEOLOGICAL STRENGTH INDEX FOR JOINTED ROCKS (Hoek and Marinos, 2000)
From the lithology, structure and surface conditions of the discontinuities, estimate the average value of GSI. Do not try to be too precise. Quoting a range from 33 to 37 is more realistic than stating that GSI = 35. Note that the table does not apply to structurally controlled failures. Where weak planar structural planes are present in an unfavourable orientation with respect to the excavation face, these will dominate the rock mass behaviour. The shear strength of surfaces in rocks that are prone to deterioration as a result of changes in moisture content will be reduced is water is present. When working with rocks in the fair to very poor categories, a shift to the right may be made for wet conditions. Water pressure is dealt with by effective stress analysis.
SURFACE CONDITIONS
VERY GOOD
Very rough, fresh unweathered surfaces
GOOD
Rough, slightly weathered, iron stained surfaces
FAIR
Smooth, moderately weathered and altered surfaces
POOR
Slickensided, highly weathered surfaces with compact coatings or fillings or angular fragments
VERY POOR
Slickensided, highly weathered surfaces with soft clay coatings or fillings
STRUCTURE
DECREASING SURFACE QUALITY
INTACT OR MASSIVE - intact rock specimens or massive in situ rock with few widely spaced discontinuities
BLOCKY - well interlocked un-disturbed rock mass consisting of cubical blocks formed by three intersecting discontinuity sets
VERY BLOCKY- interlocked, partially disturbed mass with multi-faceted angular blocks formed by 4 or more joint sets
BLOCKY/DISTURBED/SEAMY - folded with angular blocks formed by many intersecting discontinuity sets. Persistence of bedding planes or schistosity
DISINTEGRATED - poorly inter-locked, heavily broken rock mass with mixture of angular and rounded rock pieces
LAMINATED/SHEARED - Lack of blockiness due to close spacing of weak schistosity or shear planes
DECREASING INTERLOCKING OF ROCK PIECES
90
80
70
60
50
40
30
20
10
N/A
N/A
N/A
N/A