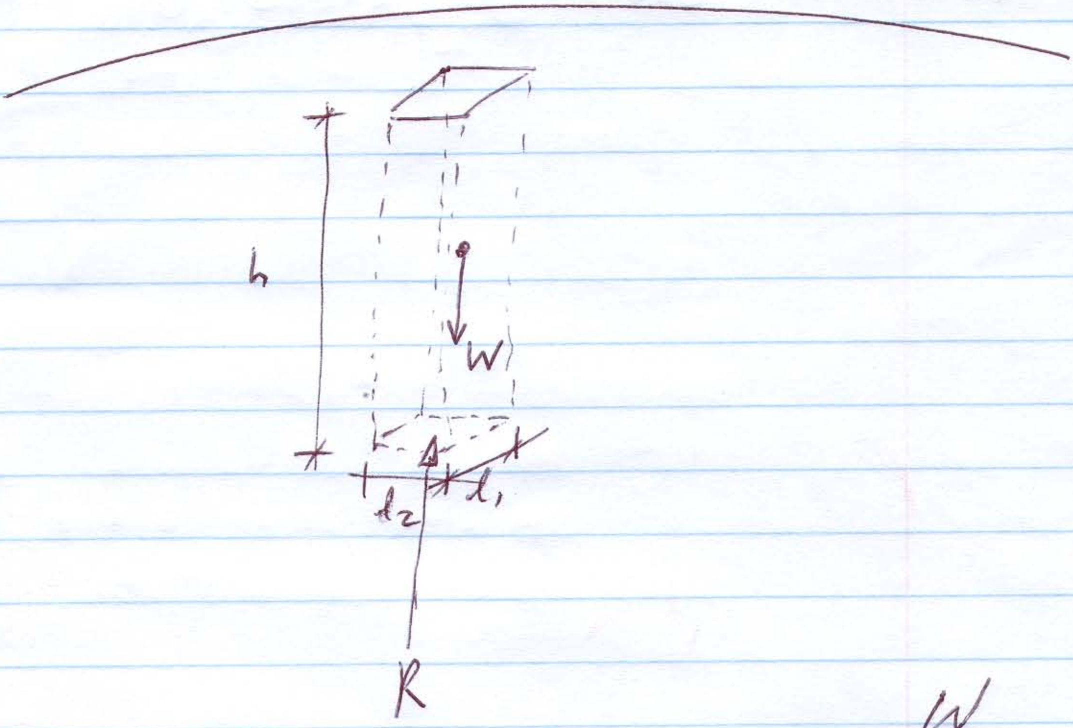
Rock Mechanics Certificate Paper 1 Model Answers

May 2007

(took me 2 hrs to do this paper, i.e. write out these 15 pages and do all the calcs).

Question 1

1.1]



The first argument is that the weight (W) of any imaginary column of rock in the Earth's crust must be exactly countered by a reaction force R. If this is not true, then the Earth's crust would be accelerating either upwards or downwards (it does on rare occasions eg in earthquakes, bombing raids etc etc etc).

If $W = R$

and $W = l_1 \times l_2 \times h \times \rho \times g$ $\left\{ \begin{array}{l} \rho = \text{rock density} \\ h = \text{depth} \\ g = \text{gravity acc} \end{array} \right.$

The vertical stress is simply the vertical force W divided by the area at the base of the column $a = l_1 \times l_2$:

$$\sigma_v = \frac{W}{l_1 \times l_2} = \frac{l_1 \times l_2 \times \rho \times g \times h}{l_1 \times l_2}$$

$$\boxed{\sigma_v = \rho g h}$$

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So, the vertical stress is directly related to the overburden weight, on average.

1.2] Variations in vertical stress arise from:
a) lateral changes in rock density, properties such as stiffness and Poisson's Ratio
b) Mining
c) Topography
d) Groundwater
e) Discontinuities and voids in the rockmass.

4

ok.

1.3]. The horizontal stresses cannot have infinite variation because rock does not have infinite strength. Therefore, a failure criterion would provide the limits to the possible horizontal stresses that can exist in the rockmass. In this instance I would choose the Hoek-Brown failure criterion, because it is based on laboratory tests of rock strength. Although the Hoek-Brown failure criterion is entirely empirical, the fundamental principal that rock has finite strength is not. So we would need some description of the rock strength to determine horizontal stress limits, and so a failure criterion like Hoek-Brown is suitable for this.

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1.4] Assumptions :- 1) We are on a continent composed mostly of SiAl rocks, hence density is 2700 kg/m^3. 2) For the H-B failure criterion assume $m = 25$, $s = 1$ (rocks are reasonably strong and intact) and UCS = 150 MPa 3) Other assumptions are that these properties are constant and that rock properties are unaffected by temperature (not true) and that even though the stresses become large, we never cross the brittle - ductile transition. By H-B:

$$\sigma_1 = \sigma_3 + \sqrt{m \sigma_c \sigma_3 + s \sigma_c^2} \quad \text{—} (1)$$

If $K < 1$, then σ_1 is vertical
$K \geq 1$, then σ_1 is horizontal.

Either σ_1 or σ_3 are known, and this is only the vertical stress $\sigma_v = \rho g h$. If $\sigma_v = \sigma_3$, then σ_1 is horizontal, and if $\sigma_v = \sigma_1$, then σ_3 is horizontal. If $\sigma_v = \sigma_3$, it's easy to use H-B, but if $\sigma_v = \sigma_1$, we have to find the minimum σ_3 from equation above, where σ_1 is known.

(4)

Rearranging H-B

$$(\sigma_1 - \sigma_3)^2 = m\sigma_c\sigma_3 + s\sigma_c^2$$

and

$$(\sigma_1 - \sigma_3)^2 - m\sigma_c\sigma_3 - s\sigma_c^2 = 0.$$

This is a quadratic equation in σ_3 for which the coefficients are given below where:-

$$a\sigma_3^2 + b\sigma_3 + c = 0$$

$$a = 1$$
$$b = -(2\sigma_1 + m\sigma_c)$$
$$c = (\sigma_1^2 - s\sigma_c^2)$$

then

$$\sigma_3 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

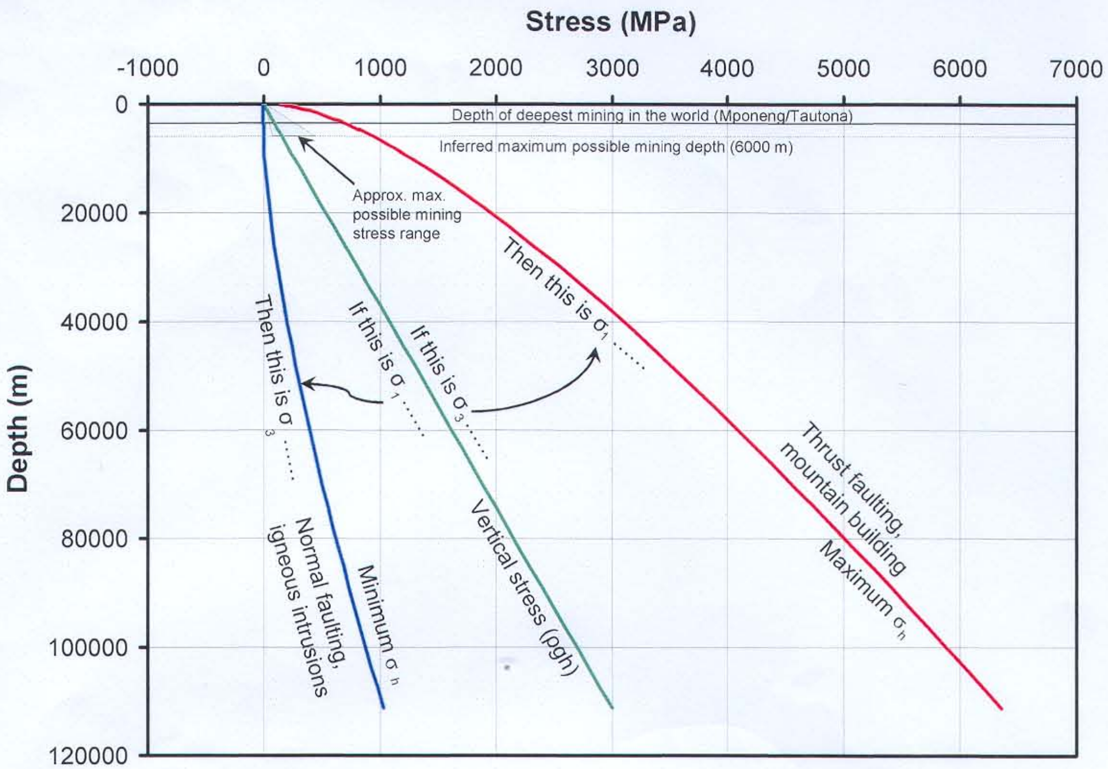
$$= \frac{2\sigma_1 + m\sigma_c \pm \sqrt{(2\sigma_1 + m\sigma_c)^2 - 4(\sigma_1^2 - s\sigma_c^2)}}{2}.$$

This will give a positive and negative root for σ_3, the negative root being meaningless because it represents a tensile stress. We want confinement (which is what we see in the crust). A table of values and a graph of the maximum and minimum stresses are attached.

P.S. Candidates are expected to calculate stresses for 3-4 depths, but as an exercise should do what is done here.

Table of maximum and minimum possible horizontal stresses

SigV (MPa)	Depth (m)	SigHmin (MPa)	SigHmax (MPa)	k (min)	k (max)
-6		-6	-6		
-5		-6	56		
-4		-6	83		
-3		-6	103		
-2		-6	120		
-1		-6	136		
0	0	-6	150		
1	37	-6	163	-5.99	163.02
2	74	-6	175	-2.99	87.60
3	111	-6	187	-1.99	62.24
4	148	-6	198	-1.49	49.41
5	185	-6	208	-1.19	41.62
6	222	-6	218	-0.99	36.36
7	259	-6	228	-0.85	32.54
8	296	-6	237	-0.74	29.64
9	333	-6	246	-0.66	27.35
10	370	-6	255	-0.59	25.49
20	741	-6	332	-0.29	16.61
30	1111	-6	397	-0.19	13.25
40	1481	-5	455	-0.14	11.38
50	1852	-5	508	-0.10	10.17
60	2222	-5	557	-0.08	9.29
70	2593	-5	604	-0.06	8.63
80	2963	-4	648	-0.05	8.10
90	3333	-4	690	-0.04	7.67
100	3704	-3	730	-0.03	7.30
110	4074	-3	770	-0.02	7.00
120	4444	-2	807	-0.02	6.73
130	4815	-1	844	-0.01	6.49
200	7407	4	1079	0.02	5.39
300	11111	16	1371	0.05	4.57
400	14815	30	1634	0.08	4.08
500	18519	48	1877	0.10	3.75
600	22222	69	2107	0.12	3.51
700	25926	92	2327	0.13	3.32
800	29630	118	2539	0.15	3.17
900	33333	146	2743	0.16	3.05
1000	37037	175	2942	0.18	2.94
1100	40741	207	3137	0.19	2.85
1200	44444	240	3327	0.20	2.77
1300	48148	274	3513	0.21	2.70
1400	51852	311	3696	0.22	2.64
1500	55556	348	3876	0.23	2.58
1600	59259	387	4054	0.24	2.53
1700	62963	426	4229	0.25	2.49
1800	66667	467	4402	0.26	2.45
1900	70370	510	4573	0.27	2.41
2000	74074	553	4743	0.28	2.37
2100	77778	597	4910	0.28	2.34
2200	81481	642	5076	0.29	2.31
2300	85185	687	5241	0.30	2.28
2400	88889	734	5404	0.31	2.25
2500	92593	782	5566	0.31	2.23
2600	96296	830	5726	0.32	2.20
2700	100000	879	5886	0.33	2.18
2800	103704	928	6044	0.33	2.16
2900	107407	979	6201	0.34	2.14
3000	111111	1029	6357	0.34	2.12



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But I don't expect an answer like this above from a candidate.

1.5]. Events and processes:

1) tectonism - folding, thrust faulting
2) volcanicity + subcrustal intrusions
3) cycles of deposition
4) cycles of erosion (5)
5) cooling and heating of rockmasses
6) wetting and drying of rockmasses.

Question 2

2.1] From the equation given and the definition of K, we can equate numerator and denominator thus:

$\sigma_h = 10 + 0.015(h - 400)$ MPa

$\sigma_v = 10 + 0.025(h - 400)$ MPa

At surface $h = 0$

$\sigma_h = 10 - 6 = 4$ MPa

$\sigma_v = 0$ MPa.

At 55 m $h = 55$ m

$\sigma_h = 10 + 0.015(-345)$ MPa $= 4.825$ MPa.

$\sigma_v = 10 + 0.025(-345)$ MPa $= 1.375$ MPa

$K = 3.51$ (not asked for).

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(8).

rel to $\theta = 45°$ line

Q 2.2] <u>Tabulated Results for 3 points</u>

$R = 3m$ and $\theta = -45°$.

Note:- I have assumed σ_1 & σ_3 for 55 m b.s. in Rei table. You

r	R/r	MPa σ_{rr}	MPa $\sigma_{\theta\theta}$	MPa $\sigma_{r\theta}$	MPa Sig1	MPa Sig2	deg $\psi°$
3	1	0.00	6.20	0.00	6.20	0.00	0
6	1/2	2.33	3.88	-2.26	5.49	0.71	36
12	1/4	2.91	3.29	-1.92	5.03	1.17	42

Can take reducing depth into account, but this will complicate calcs.

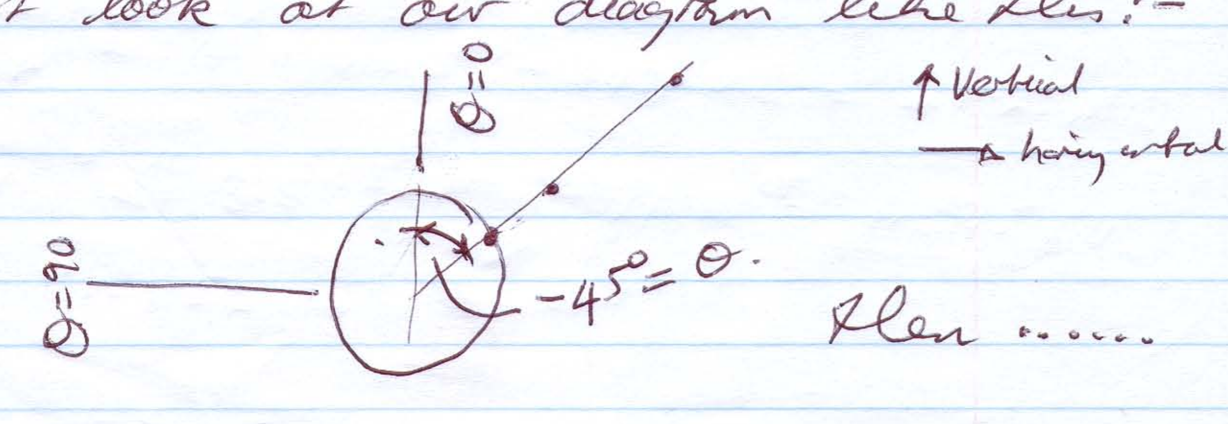
Use the Kirsch equations to find σ_{rr}, $\sigma_{\theta\theta}$ and $\sigma_{r\theta}$. These are <u>NOT</u> the principal stresses in general because they are accompanied by ($\sigma_{r\theta} \neq 0$) shear stresses. You then have to find the principal stresses by

$$\sigma_{1,2} = \tfrac{1}{2}(\sigma_{rr} + \sigma_{\theta\theta}) \pm \sqrt{(\sigma_{rr} - \sigma_{\theta\theta})^2 + 4\sigma_{r\theta}^2}$$

and $$\psi = \frac{1}{2}\tan^{-1}\left[\frac{2\tau_{r\theta}}{\sigma_{rr} - \sigma_{\theta\theta}}\right].$$

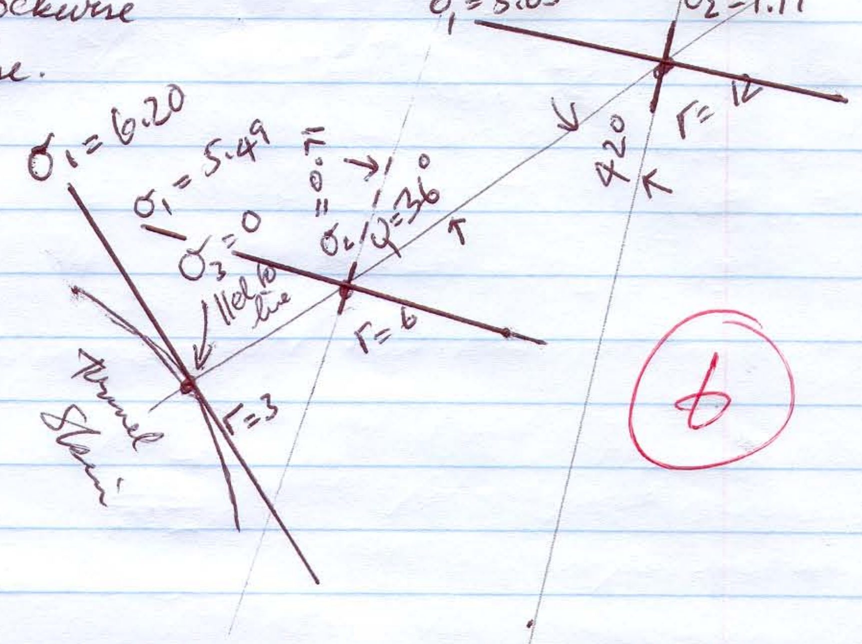
12.

2.3] Now ψ is given <u>in relation to the line</u> along which you have been calculating the stresses, in this case at 45° to the horizontal. The Kirsch solution also assumes that σ_2 = horiz, and σ_1 is vertical. Since we have stresses the other way around, we must look at our diagram like this:-



then .....

ψ defines the angle between σ_2 and the line.
Positive anti-clockwise
Negative clockwise.



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2.4]. Calculate $\sigma_{\theta\theta}$ for $\theta = 0°$ (ie at the top of the tunnel); and $r = 3$ m.

$$\sigma_{\theta\theta} = 13.10 \text{ and } \sigma_{rr} = 0.00 \text{ MPa.}$$

By H-B

$$\sigma_{1(max)} = 0 + \sqrt{0 + 1 \times 30^2}$$

$$= 30 \text{ MPa}$$

Therefore the tunnel will not fail in the roof.

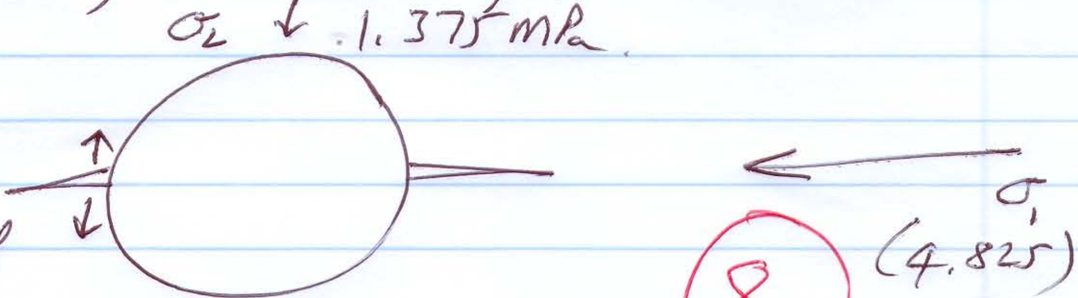
In the sidewall there is a tensile vertical stress. Is the sandstone strong enough to withstand this? To find out, we make $\sigma_1 = 0$, then

$$\sigma_3^2 - M\sigma_c\sigma_3 \mp S\sigma_c^2 = 0.$$

$$\sigma_3 = \frac{M\sigma_c \pm \sqrt{M^2\sigma_c^2 + 4S\sigma_c}}{2}$$

$$= \frac{12 \times 30 \pm \sqrt{144 \times 900 + 4 \times 900}}{2} = -2.48 \text{ MPa}$$

According to the H-B criterion, the rock has a tensile strength of -2.48 MPa (the other root is meaningless because it gives a large positive answer). In the tunnel sidewall ($\theta = 90°$) we find a vertical tensile stress parallel to the sidewall of -0.7 MPa. This means we do not expect tensile failure wherever the sandstone conforms to the H-B strength parameters. However, where it doesn't, we could get horizontal fractures developing into the sidewall along weak joints, or bedding.



Horizontal tensile splits. Cracks open vertically.

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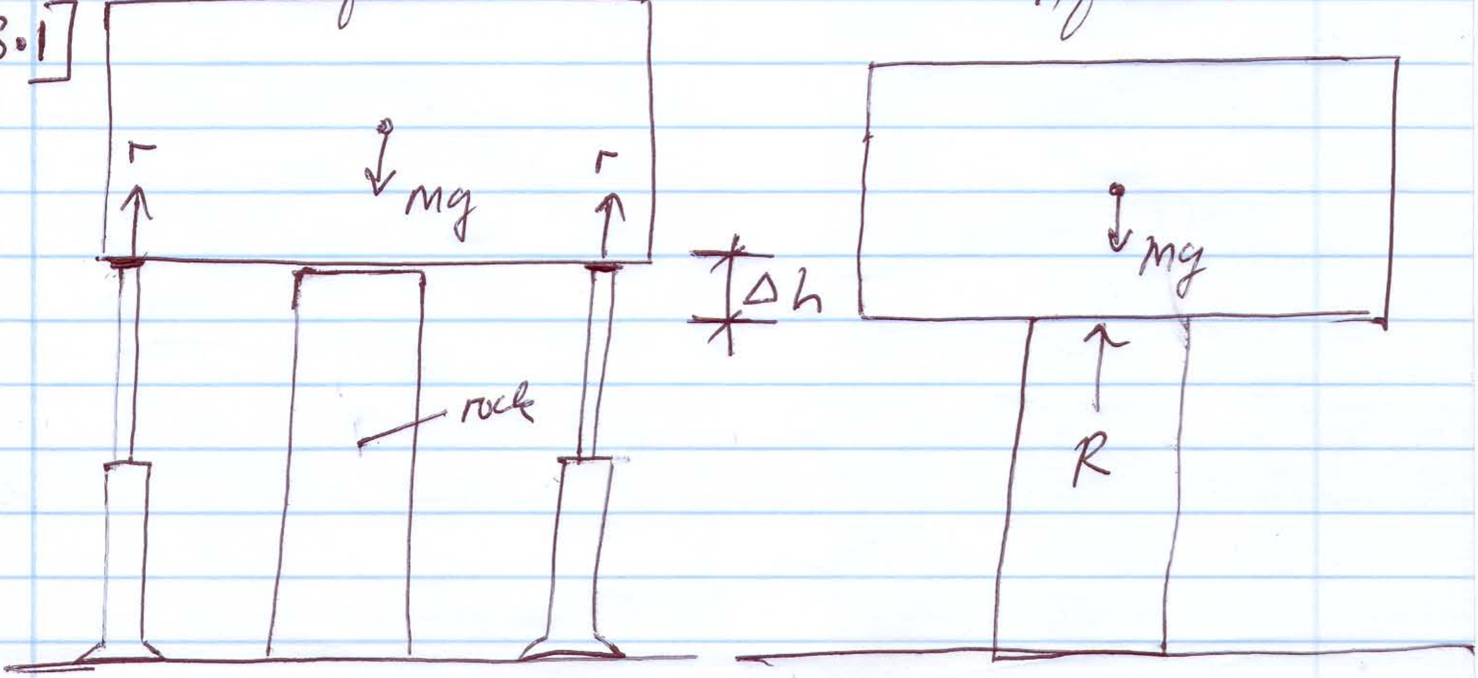
Question 3

3.1

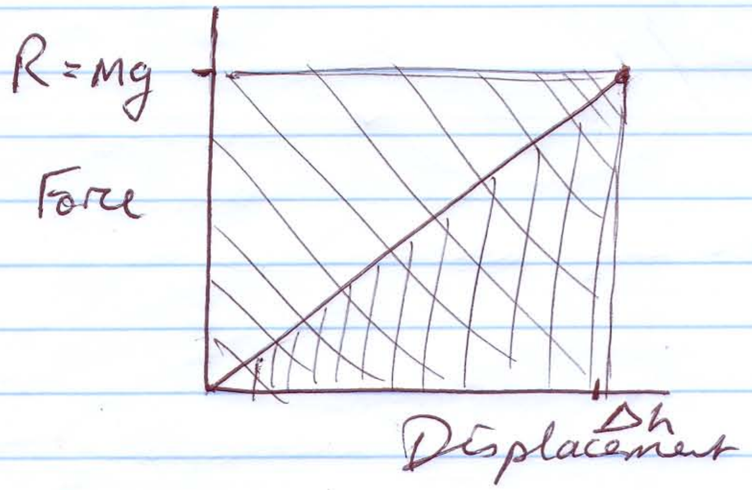


In first case, Mass Mg is supported by props just above rock block. The jacks are then lowered and the mass is allowed to settle on the rock, compressing it by a distance Δh, when the props are removed.

Energy changes in system

$$\Delta(\text{Potential Energy of mass}) = Mg\Delta h$$

$$\Delta(\text{Strain energy in rock}) = \frac{1}{2} R \Delta h.$$



Energy change in system $= Mg\Delta h$

Energy stored in rock $= \frac{1}{2} R\Delta h$.

Therefore Energy released (or lost) $= Mg\Delta h - \frac{1}{2} R\Delta h$

$$= \frac{1}{2} Mg\Delta h = \frac{1}{2} R\Delta h.$$

This is known as the energy released, and in the case of mines it is the difference between the loss of potential energy in the rockmass minus the ~~extra~~ strain energy stored in the rockmass when a mining step (blast) is taken.

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(12)

3.2] $G = \frac{E}{2(1+\nu)} = \frac{50}{2(1+0.25)} = \underline{33.33 GPa}$

Vertical Stress $q = \rho g h = 3000 \times 9.81 \times 1500$
$= \underline{44.15 MPa}$

Face stress is 264.87 MPa. (6×44.15).

$$S_z = \frac{2(1-0.25)\,44.15}{33333}\sqrt{51^2 - 50.5^2} \text{ m.}$$

$= \underline{0.014 \text{ m.}}$

ERR = ½ (Stress before mining * convergence after mining)

$= \frac{1}{2} * 264.87 * 0.014$ MJ/m²

$= \underline{1.87 \text{ MJ/ca.}}$ 6.

3.3]. Convergence when $l = 50m$:

$$S_z = \frac{2(1-0.25)\,44.15}{33333}\sqrt{50^2 - 0^2}$$

$= 0.099 \text{ m} = \underline{99 \text{ mm}}$

S_z after the blast when $l = 51m$:

$$S_z = \frac{2(1-0.25)\,44.15}{33333}\sqrt{51^2 - 0^2}$$

$= 0.101 \text{ ~~mm~~} = \underline{101 \text{ mm}}$ 4

∴ Expect 2 mm convergence in ~~Centre~~ gully after blast.

3.4] The stope will be closed 50% when $S_z = 500mm = 0.5m$.

$$\therefore\ 0.5 = \frac{2(1-0.25)44.15}{33333}\sqrt{l^2}$$

$$\text{or } l = \frac{0.5 \times 33333}{2(1-0.25)44.15}$$

$$= 251.66 \cong 252\,M$$

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Since l is already at 51 m, and each blast gives an advance of 1m, we can expect to take another 201 blasts approximately.

Question 4

4.1]. A boundary condition in the class of boundary value problems is ~~either~~ a specification of the stress state or strain state on the boundary of a body, or equivalent force or displacement specification.

eg 1) A fixed boundary in y-direction. This is a <u>displacement</u> boundary condition - boundary (hence body) can move in X, but not in y.

2) A force boundary condition, where a force is specified to be acting on the body boundary, eg the weight of a house on its foundation. The foundation would be replaced by the force on the soil boundary.

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4.2] Plane stress exists when all normal and sheer stresses associated with a plane are <u>zero</u>.

1). The earth's surface is in a plane state of stress, since there are no vertical stresses or sheer stresses acting on the surface.

2) The hull of a boat, aeroplane or car is in a ~~pla~~ state of approximate plane stress if the thickness of the structure is small in relation to its size.

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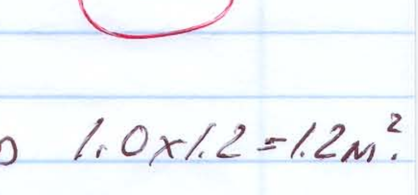
4.3]. Fallout height 1.8m
Area supported by each prop $1.0 \times 1.2 = 1.2\,m^2$.

∴ Volume of rock supported by each prop $= 2.16\,m^3$
and Mass of rock = 5832 kg.

If we wish to decelerate 5832 kg of rock from 3.0 m/s to 0 m/s in the space of 0.2 m, and simple Newtonian Mechanics:

$$(V^2 - V_0^2) = 2a(x - x_0)$$



$$0 - 3^2 = 2a * 0.2\,m$$

$$\text{or} \quad a = \frac{9}{0.4} = -22.5\,m/s^2.$$

Now, in addition to this acceleration, we have to include the acceleration due to gravity (which will give a negative value, because it is deceleration).

$$a = -32.31 \, m/s^2$$

$F = m\,|a|$ (we want to know the force upwards, not the deceleration downwards).

$$= 5832 \times 32.31 = 188.43 \, kN.$$

Therefore each prop has to provide 188.43 kN yielding force $\sim$ 20t.

Now, normalizing this to support resistance per square metre:-

$$\frac{188.43}{1.2} = 157 \, kN/m^2$$, which

is about 4 x the requirement for a rockfall problem only.

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100 Marks