(1).

MODEL ANSWERS ROCK MECHANICS PAPER I

25 October 2004

Question 1a

(10) Marks

1a) $\varepsilon_{xx} = 0.677e-3$, $\varepsilon_{xy} = 0.550e-3$, $\varepsilon_{yy} = 1.120e-3$

There are two ways to do this problem, first calculate the stresses from the strain and then find the principal stresses, or find the principal strains from the above strain, and then calculate the principal stresses directly from the principal strains. This memorandum will follow the first-mentioned route. The student is encouraged to rework the problem using both routes.

$$\sigma_{xx} = \lambda\Delta + 2G\varepsilon_{xx}$$
$$= 25.96e9 \times 1.797e-3 + 2\times 17.31e9 \times 0.677e-3$$
$$\sigma_{xx} = 70.09 \text{ MPa}$$

$$\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)} = \frac{0.3\times 45}{(1+0.3)(1-0.6)} = 25.96 \text{ GPa}$$

$$\sigma_{yy} = \lambda\Delta + 2G\varepsilon_{yy} = 46.65 + 38.77$$
$$\sigma_{yy} = 85.42 \text{ MPa}$$

$$\Delta = \varepsilon_{xx} + \varepsilon_{yy} \quad (\text{assume } \varepsilon_{zz} = 0)$$
$$= (0.677 + 1.120)e-3$$
$$\Delta = 1.797e-3$$

$$\sigma_{xy} = G\gamma_{xy} = 2G\varepsilon_{xy}$$
$$= 2\times 17.31 \times 0.55e-3$$
$$= 19.04 \text{ MPa}$$

$$G = \frac{E}{2(1+\nu)} = \frac{45}{2(1+0.3)} = 17.31 \text{ GPa}$$

(2).

Note in the above we have implicitly assumed plane strain, because we have assumed ε_{zz} in Δ to be $\varepsilon_{zz} = 0$. Is this correct? Well, let us first assume plane strain and then rework the problem assuming plane stress.

If we assume plane strain, then

$$\sigma_{xx} = 70.09 \text{ MPa}, \quad \sigma_{xy} = 19.04 \text{ MPa}, \text{ and}$$

$$\sigma_{yy} = 85.42 \text{ MPa}.$$

Finding the principal stresses:-

$$\sigma_{1,2} = \tfrac{1}{2}\left(\sigma_{xx} + \sigma_{yy}\right) \pm \tfrac{1}{2}\sqrt{\left(\sigma_{xx} - \sigma_{yy}\right)^2 + 4\sigma_{xy}^2}$$

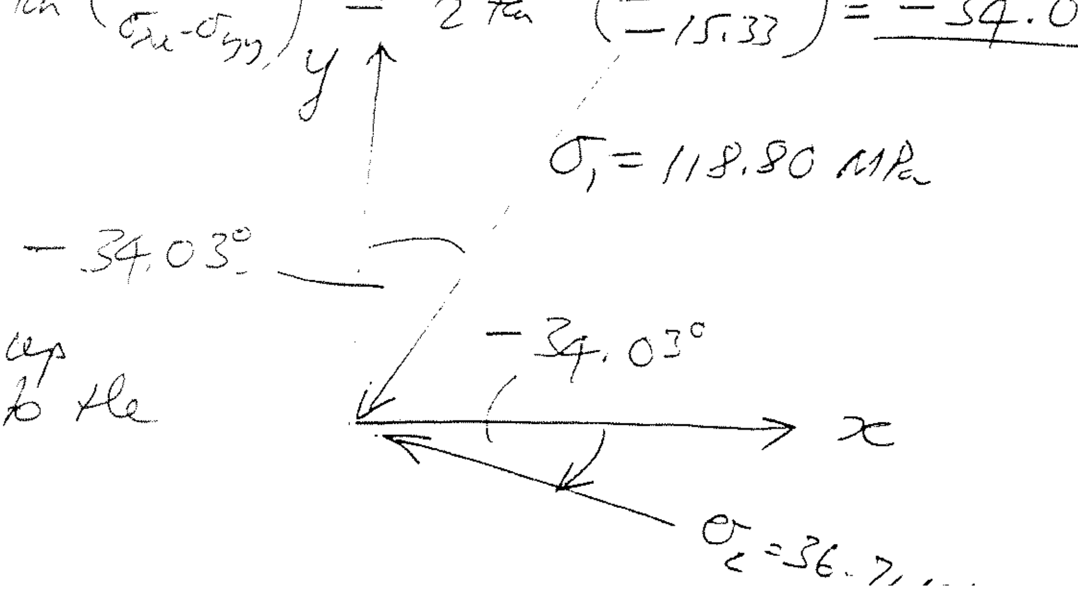
$$\Rightarrow \tfrac{1}{2}\sqrt{(70.09 - 85.42)^2 + 4\,(19.04)^2} = 41.05 \text{ MPa}.$$

$$\sigma_1 = \tfrac{1}{2}\left(\sigma_{xx} + \sigma_{yy}\right) + 41.05 = \underline{118.80 \text{ MPa}}$$

$$\sigma_2 = \tfrac{1}{2}\left(\sigma_{xx} + \sigma_{yy}\right) - 41.05 = \underline{36.71 \text{ MPa}}$$

$$\theta = \tfrac{1}{2}\tan^{-1}\left(\frac{2\sigma_{xy}}{\sigma_{xx} - \sigma_{yy}}\right) = \tfrac{1}{2}\tan^{-1}\left(\frac{38.08}{-15.33}\right) = \underline{-34.03^\circ}$$

Looking up borehole to the end:



10 Marks

1b) If the borehole is 2000m below surface, then

Vertical stress due to overburden weight $= \rho g h$

$= 3000 \times 9.81 \times 2000$

$\sigma_{ob} = 58.86$ MPa, vertical

If the y-axis is vertical, then this must be compared with $\sigma_{yy} = 85.42$ MPa.

The measured stress of 85.42 MPa is 45% larger than the expected overburden stress. Now, such a measurement could be locally possible due to either tectonic disturbance or a geological structure. If there is no external evidence that this is possible, then the measurement is likely to be incorrect. In fact, the measurement (which is purely a strain-relief measurement) may not be incorrect – it is the assumption of PLANE STRAIN when computing the associated stresses that is incorrect.

Mechanically plane strain conditions probably exist in only rare circumstances (especially in nature) but plane stress is far more common. For example every free surface in a solid is in a plane stress state, as is the flattened end of a borehole.

4.

Reworking the problem using plane stress, we have to assume that $\sigma_{zz} = 0$. Using the equations of elasticity:

$$\varepsilon_{zz} = \frac{1}{E}\left(\sigma_{zz} - \nu(\sigma_{xx} + \sigma_{yy})\right)$$

$$\Rightarrow \varepsilon_{zz} = \frac{-\nu}{E}(\sigma_{xx} + \sigma_{yy}) = \frac{-0.3}{45000}(\sigma_{xx} + \sigma_{yy}).$$

Now,

$$\sigma_{xx} = \lambda\Delta + 2G\varepsilon_{xx} \qquad \lambda = 25.96\ GPa$$

$$\sigma_{xx} = 25960\left(\varepsilon_{xx} + \varepsilon_{yy} - \frac{0.3}{45000}(\sigma_{xx} + \sigma_{yy})\right) \qquad \Delta = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}$$

$$+ 2 \times 17310 \times 0.677e{-3}\ MPa \qquad G = 17.31\ GPa$$

$$\sigma_{xx} + 0.1731\,\sigma_{xx} + 0.1731\,\sigma_{yy} = 70.09\ MPa$$

$$1.1731\,\sigma_{xx} + 0.1731\,\sigma_{yy} = 70.09\ MPa \quad \text{---- } (1)$$

$$\sigma_{yy} = 25960\left(\varepsilon_{xx} + \varepsilon_{yy} - \frac{0.3}{45000}(\sigma_{xx} + \sigma_{yy})\right)$$

$$+ 2 \times 17310 \times 1.120e{-3}\ MPa$$

$$\Rightarrow 1.1731\,\sigma_{yy} + 0.1731\,\sigma_{xx} = 85.42\ MPa \quad \text{---- } (2)$$

Multiplying equation (2) by $\frac{0.1731}{1.1731}$

$$0.1731\,\sigma_{yy} + 0.0255\,\sigma_{xx} = 12.60\ MPa \quad \text{--- } (3)$$

Subtracting (3) from (1)

$1.1476\,\sigma_{xx} = 57.49$ MPa

$\Rightarrow \underline{\sigma_{xx} = 50.09 \text{ MPa}}$

Hence $\underline{\sigma_{yy} = 65.42 \text{ MPa}}$

$\underline{\sigma_{xy} = 19.04 \text{ MPa}}$ (Same as before)

The simplest way of finding the stresses is to implicitly assume that $\sigma_{zz} = 0$ in the equations of elasticity:

$$\varepsilon_{xx} = \frac{1}{E}\left(\sigma_{xx} - \nu\sigma_{yy}\right)$$

$$0.677e{-3} = \frac{1}{45000}\left(\sigma_{xx} - 0.3\sigma_{yy}\right)$$

$$\Rightarrow \sigma_{xx} - 0.3\sigma_{yy} = 30.465 \text{ MPa} \quad \text{----} \quad (1)$$

Similarly

$$\sigma_{yy} - 0.3\sigma_{xx} = 50.400 \text{ MPa} \quad \text{----} \quad (2)$$

Multiplying equation (2) by 3.333 and adding result to (1):

$$3.033\,\sigma_{yy} = 198.448 \text{ MPa}$$

$$\left.\begin{aligned} &\Rightarrow \underline{\sigma_{yy} = 65.43 \text{ MPa}} \\ &\text{and } \underline{\sigma_{xx} = 50.10 \text{ MPa}} \end{aligned}\right\} \text{Almost identical to answers above.}$$

$\underline{\sigma_{xy} = 19.04 \text{ MPa}}$ Same as before.

Comparing the vertical virgin stress $\rho g h$ due to overburden weight we have

$$\sigma_{virgin} = \rho g h = 3000 \times 9.81 \times 2000 = 58.86\ MPa,$$

which must be compared with 65.43 MPa,

turns out to have an error of 11.2%, far more acceptable than the error of some 45% obtained earlier. Therefore the assumption of <u>PLANE STRESS</u> which is a mechanically more acceptable assumption for the free surface at the end of the borehole, results in a better measurement result <u>with the same strain relief measurements</u>.

This discourse is intended to highlight the following points :-

- Valid assumptions lead to better results
- there are many ways to solve the problem, some computationally simpler than others.
- the equations are consistent i.e they give the same results
- the stress result is not exactly in agreement with the stress due to the overburden weight, which is typical of reality, and the error depends entirely on the initial assumptions.

What about the principal stresses in the plane stress case? They will have different magnitudes and the same directions. The student is encouraged to verify this result.

Question 1(c).

From what has gone before, strain reliefs from a single borehole are <u>not</u> sufficient to determine the in-situ stress state in a rockmass. The reason for this is that the rockmass is 3-dimensional, as is the stress tensor extant in the rockmass 3-dimensional. Hence making a 2-dimensional measurement and then assuming either plane stress or plane strain can never give the full 3-D picture. Therefore if one is going to go to the trouble of measuring in-situ stresses from strain relief results, then there is no sense in obtaining only 2-D strain reliefs, and every sense in obtaining 3-D strain reliefs.

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Question 2

Question 2 (a)

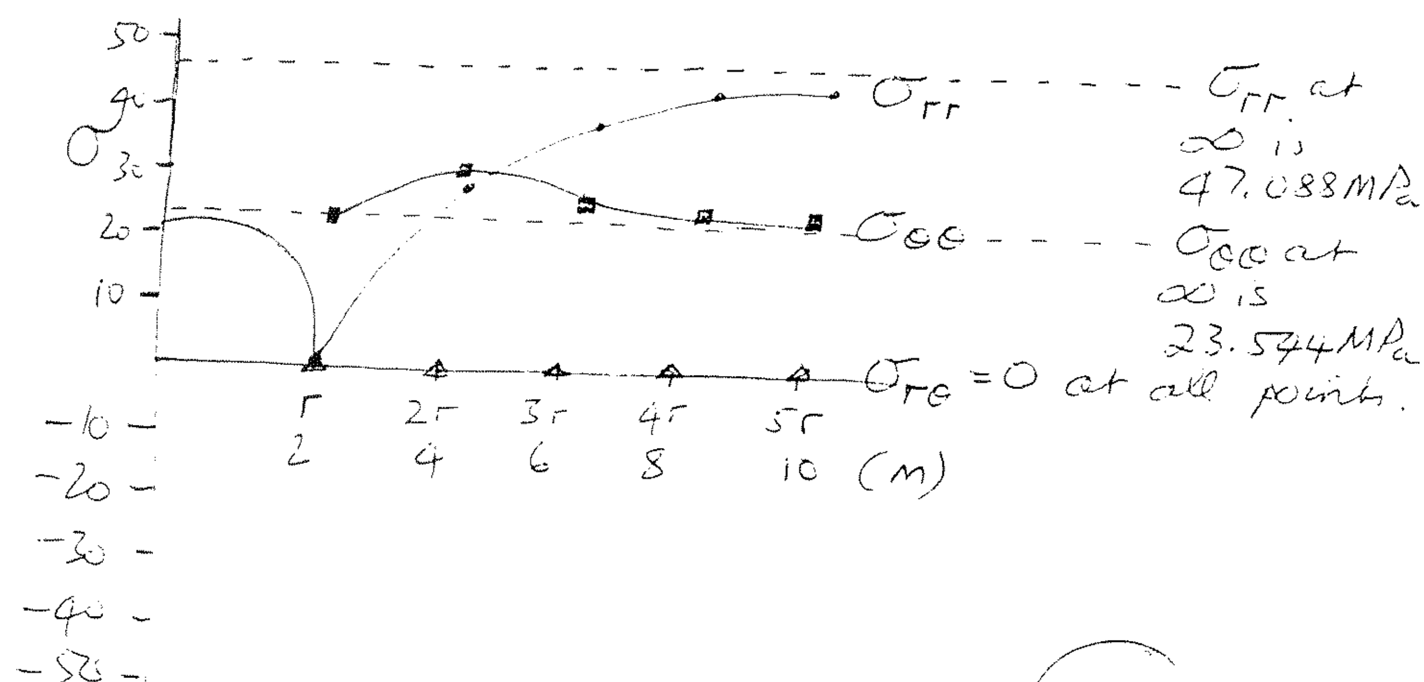
$\sigma_v = \rho g h = 3000 \times 800 \times 9.81 = \underline{23.544 \text{ MPa}}$

$\sigma_h = 2\sigma_v = \underline{47.088 \text{ MPa}}$

Applying the Kirsch equations and tabulating the results:–

$R = 2.0$ m

r	σ_{rr} (MPa)	$\sigma_{\theta\theta}$ (MPa)	$\sigma_{r\theta}$ (MPa)
2.0	0.00	23.54	0.0
4.0	28.69	30.17	0.0
6.0	38.37	27.03	0.0
8.0	42.08	25.61	0.0
10.0	43.85	24.90	0.0



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Question 2 (b)

$E = 60$ GPa $\quad \nu = 0.30$.
$R = 2.0$ m, and using equation of elasticity:

r	ε_{rr}	$\varepsilon_{\theta\theta}$	$\varepsilon_{r\theta}$
2.0	-0.000118	0.000392	0.000000
4.0	0.000327	0.000359	0.000000
6.0	0.000504	0.000259	0.000000
8.0	0.000573	0.000217	0.000000
10.0	0.000606	0.000196	0.000000

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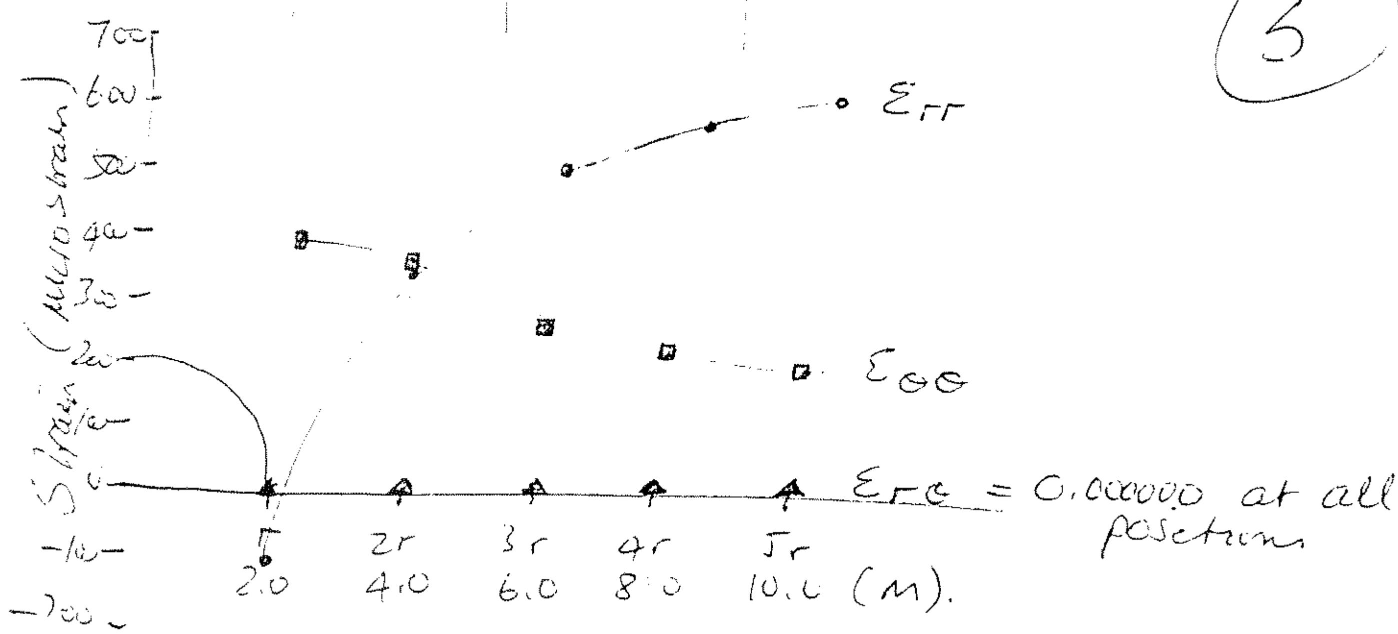

Question 2 (c)

-0.2 millistrain is -0.000200 strains $= -200$ microstrains

According to the criterion, there is ~~no~~ NO failure because it is not satisfied at any point along the profile calculated. Hence there is NO spalling - tunnel remains intact.

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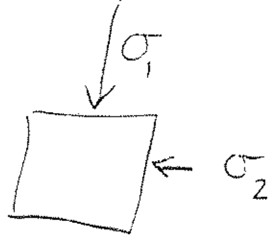
Question 2(d).

The simplest way to demonstrate this is to consider a rockmass in compression in 2-D, where $\sigma_1 \gg \sigma_2$, but both σ_1 and σ_2 are compressive stresses.



From the equations of elasticity :-

$$\varepsilon_1 = \frac{1}{E}(\sigma_1 - \nu\sigma_2) \quad \text{--- (1)}$$ (We are assuming plane stress)

$$\varepsilon_2 = \frac{1}{E}(\sigma_2 - \nu\sigma_1) \quad \text{--- (2)}.$$

If $\sigma_1 \gg \sigma_2$, this difference has to be large enough such that $\nu\sigma_1 > \sigma_2$. Then ε_2 will be negative, i.e. a tensile strain. In a uniaxially compressed rock specimen, ε_2 (horizontal) has to be tensile, because σ_2 (horizontal) is zero.

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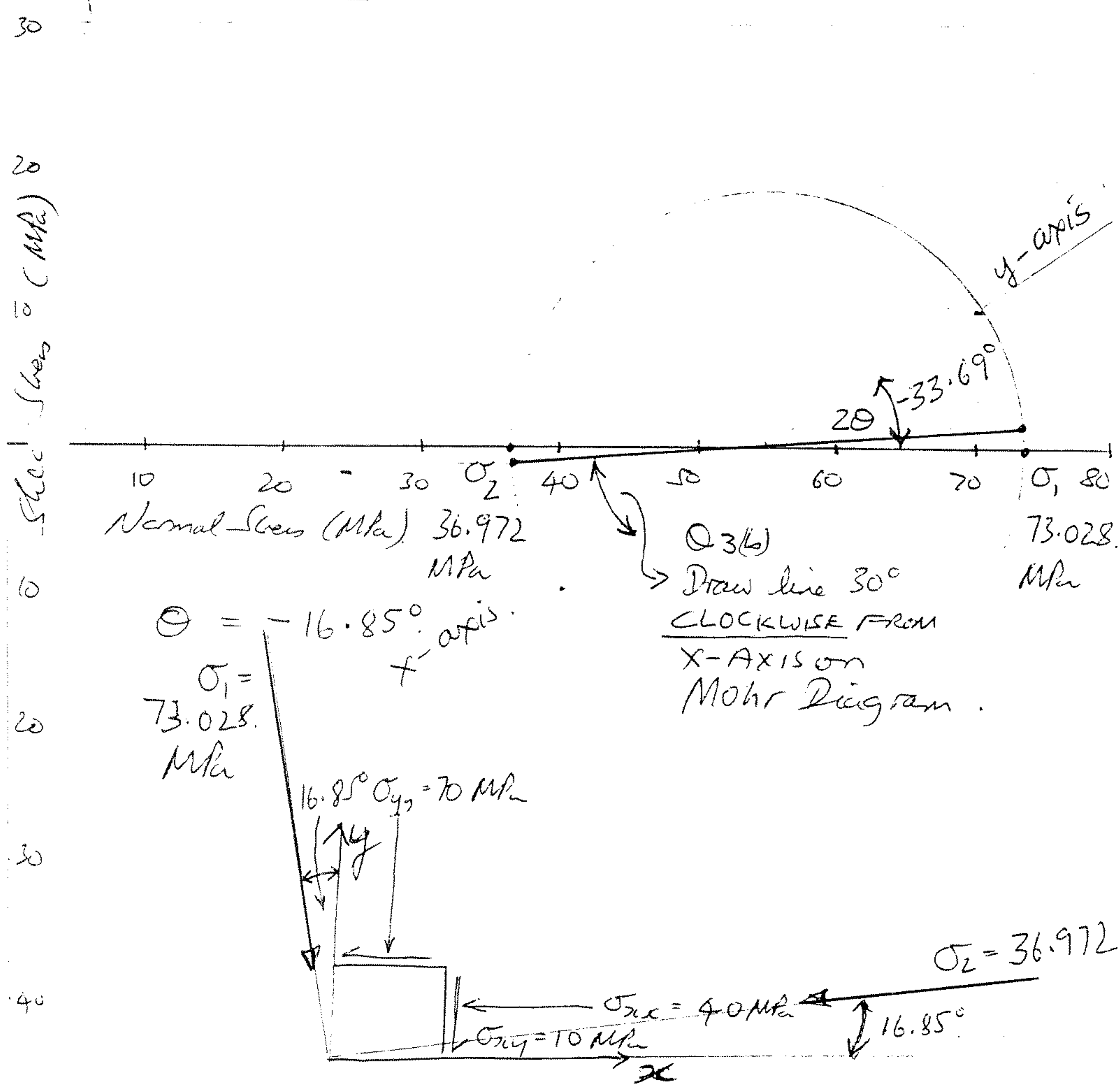
Question 3



Question 3(a) $\sigma_1 = 73.028$ MPa
$\sigma_2 = 36.972$ MPa
$\theta = \underline{\underline{+106.85°}}$ (It is the angle that σ_1 makes with the x-axis)

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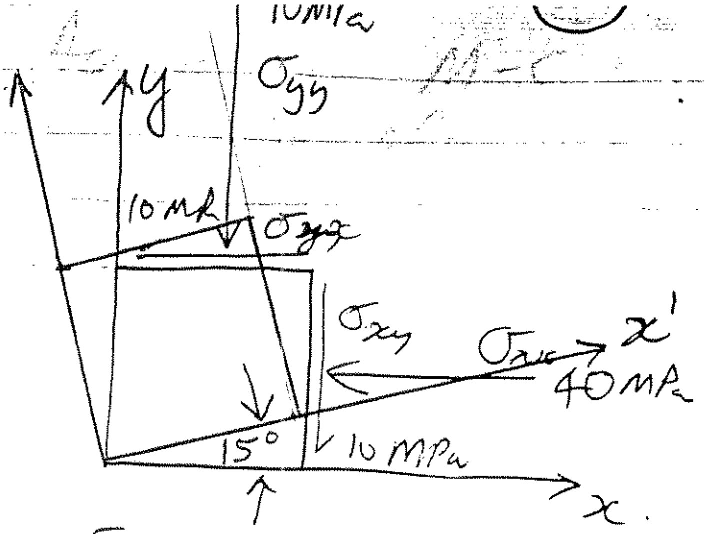
Question 3(b) Estimate $\sigma' = 73$ MPa
$\sigma'_{\cancel{yy}\,xx} = 37$ MPa
$\sigma'_{xy} = 1$ MPa.



The plane whose normal makes a 15° angle with the X axis is the one that σ'_{xx} and σ'_{xy} act on. In terms of the Mohr Circle angle = −15°.

From Equation

$$\sigma'_{xx} = \sigma_{xx}\cos^2\theta + 2\sigma_{xy}\sin\theta\cos\theta + \sigma_{yy}\sin^2\theta$$

$$= 40\cos^2 15 + 20\sin 15\cos 15 + 70\sin^2 15.$$

$$= 37.32 - 5.00 + 4.69$$

$$= 37.01 \text{ MPa}$$

$$\sigma'_{xy} = \frac{1}{2}(\sigma_{yy} - \sigma_{xx})\sin -30 + \sigma_{xy}\cos -30.$$

$$= -7.5 + 8.66$$

$$= +1.16 \text{ MPa}$$

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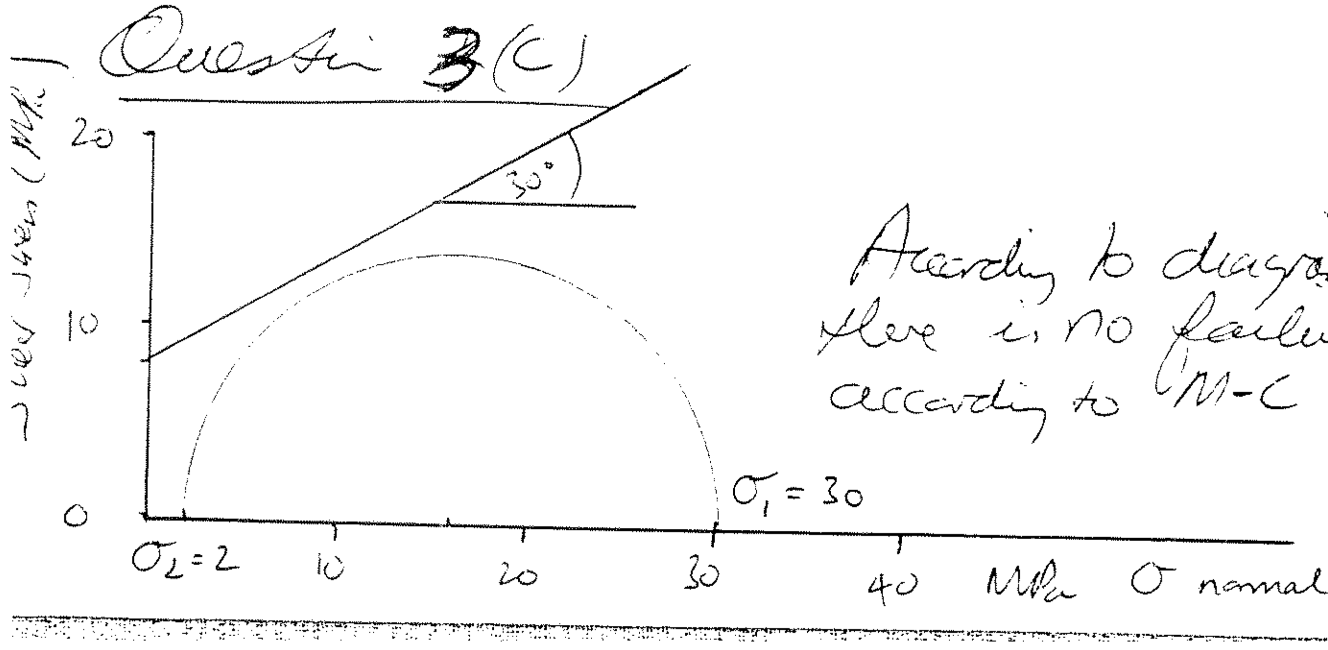
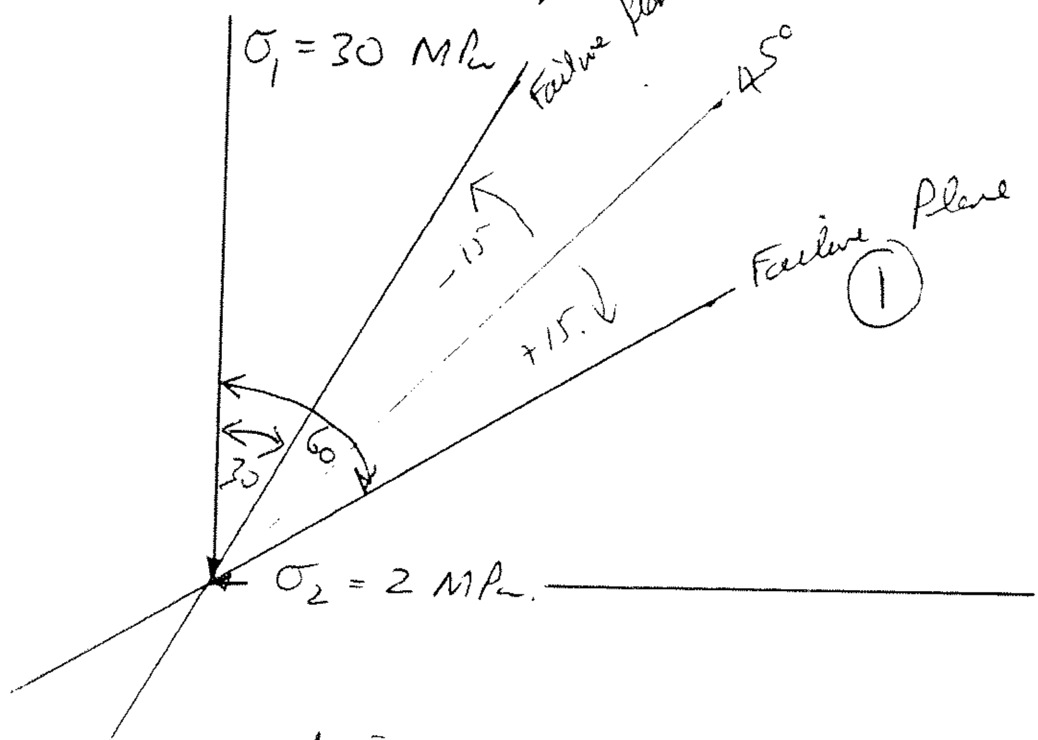
Question 3(C)



According to diagram there is no failure according to M-C diagram.

According to M-C criterion, failure in shear will take place along two conjugate planes given by $45° \pm \phi/2$. If $\phi = 30°$, then the conjugate planes are 30° and 60° wrt the maximum principal stress.



For $\theta = 30°$ — ie failure plane ①.

$$\sigma'_{xx} = \sigma_2 \cos^2\theta + \sigma_1 \sin^2\theta$$

$$= 2\cos^2 30 + 30 \sin^2 30 = \underline{9 \text{ MPa}}$$

$$\sigma'_{yy} = \sigma_2 \sin^2\theta + \sigma_1 \cos^2\theta = \underline{23 \text{ MPa}}$$

$$\sigma'_{xy} = \tfrac{1}{2}(\sigma_1 - \sigma_2)\sin 60 = \underline{12.1244 \text{ MPa}}$$

Now, the clamping force on failure plane ① we must use $\sigma'_{yy} = 23$ MPa.

Applying M.C. $|\tau| - (C_0 + \mu\,\sigma_n) \geq 0$ for failure

$12.1244 - (0 + \tan 30 \times 23) = 12.12 - 13.28$ MPa.

Hence there cannot be failure on the first conjugate plane. By symmetry of the stress rotation equations the conditions on the second conjugate plane are identical, but this time σ'_{xx} is the clamping force and the conditions are the same: no failure, in agreement with the diagram.

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Question 3 (d).

$$\nu = -\frac{\varepsilon_l}{\varepsilon_a},$$ where ε_l = lateral strain = $\frac{\Delta r}{r_0}$

where r = specimen radius

and ε_a = axial strain = $\frac{\Delta l}{l_0}$

Using the engineering sign convention,

$$\varepsilon_a = -\frac{\Delta l}{l_0} \quad \text{(Compression results in shortening)}$$

and $$\varepsilon_l = \frac{\Delta r}{r_0} \Rightarrow \nu = -\frac{(\Delta r / r_0)}{(-\Delta l / l_0)}$$

$$= \frac{|\Delta r \, l_0|}{|\Delta l \, r_0|} \quad — 1$$

$$V = \pi l_0 r_0^2$$

$$V + \Delta V = \pi (l_0 - \Delta l)(r_0 + \Delta r)^2$$

$$\therefore \Delta V = \pi \left[(l_0 - \Delta l)(r_0 + \Delta r)^2 - l_0 r_0^2 \right] \quad — 2$$

Now in a typical ISRM Uniaxial test $H = 2.5D$ (height = 2.5 diameter), hence $l \doteq 5r$. Also, a typical Poisson's Ratio for rock is 0.2.

Hence from equation 1

$$0.2 = \frac{\Delta r \; 5 r_0}{\Delta l \; r_0}$$

or $|\Delta r| = |0.04 \Delta l|$ or $|\Delta l| = |25 \Delta r|$

Putting this into equation 2

$$\Delta V = \pi \left[(5r_0 - 25\Delta r)(r_0 + \Delta r)^2 - 5r_0^3 \right]$$

$$= \pi \left[(5r_0 - 25\Delta r)(r_0^2 + 2r_0\Delta r + \Delta r^2) - 5r_0^3 \right]$$

$$= \pi \left[\cancel{5r_0^3} + 10r_0^2\Delta r + 5r_0\Delta r^2 - 25\Delta r\, r_0^2 - 50\Delta r^2 r_0 - 25\Delta r^3 - \cancel{5r_0^3} \right]$$

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$$= \pi \left[-15r_0^2\Delta r - 45r_0\Delta r^2 - 25\Delta r^3 \right]$$

Now, if Δr is a positive quantity, i.e. an <u>increase</u> in rock specimen diameter, then the ΔV given above for a typical UCS test with $\nu = 0.2$ will result in a <u>negative</u> volume change in the elastic region. $\Delta V < 0$.

Question 4(a)

Textbook 2

Question 4(b)

Textbook 2

Question 4(c)

Textbook 2.

Question 4(c)

Textbook 2

Question 4(d)

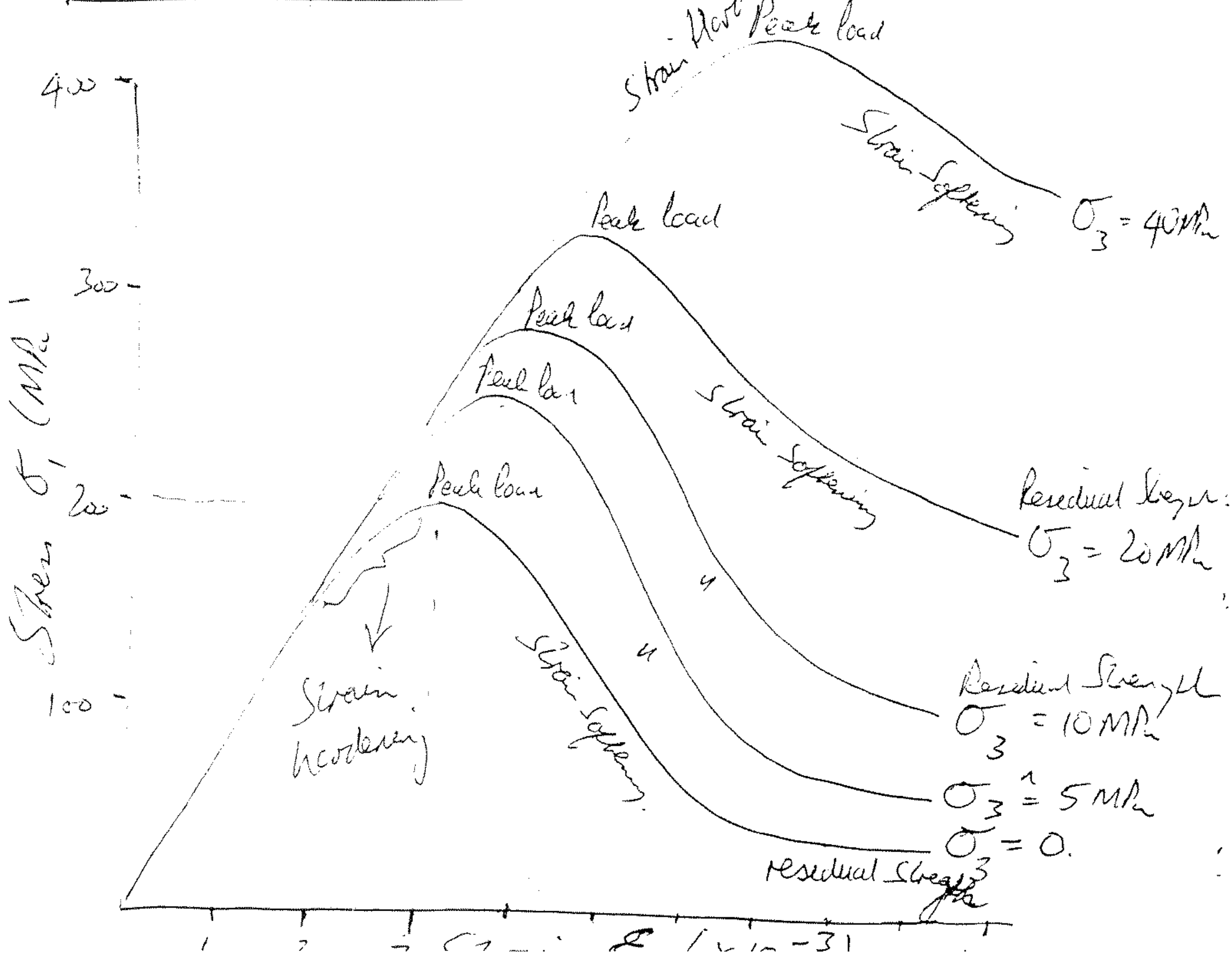
Textbook 4

Q 4(g) $\sigma_1 = \sigma_3 + \sqrt{m\sigma_c\sigma_3 + s\sigma_c^2}$

$= 20 + \sqrt{15 \times 200 \times 20 + 1 \times 200^2}$

$= 336.228$ MPa

Question 4(f).



Question 5 – this is a judgemental question – these are my answers which are not ~~necess~~ necessarily absolutely correct.

(a) To measure and record any rock parameter with the purpose of using the information for planning and design purposes. 2

(b) 1). To compare expected response of rockmass to measured response.
2) To provide input ~~foto~~ numerical and analytical models.
3) To provide input for planning. 8.
4) To provide input for design.

(c) 1) Rock fracture – monitor intensity, change in intensity, direction and whether opening or not (extensometers) to determine state of excavation and for input into numerical models to determine whether excavation is performing to expectations, and whether any assumptions such as virgin stress state, induced stress changes, rock strength, rock structure or unknown geology have had an influence. Mainly visual measurements and some extensometer measurements.
2) Closure. For similar reasons named above. Extensometers, survey peg positions, closure meters.
3) Rockmass condition. Is it weathering and deteriorating in strength due to exposure to air and water? Rock strength tests, visual inspections.
4) Virgin stress (strain relief measurements) followed by mining induced stress change measurements

measured by hard inclusion stressmeters or soft inclusion strain measurement devices or by numerical modelling.

5) Support condition: tell-tales, tension in tendons, dryrete/wetrete condition, closure measurements on support elements.

Some or all of these should be used to monitor large vital excavations in mines in the long term.

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