Rock Mechanics Paper 1 Model Answers
October 2006

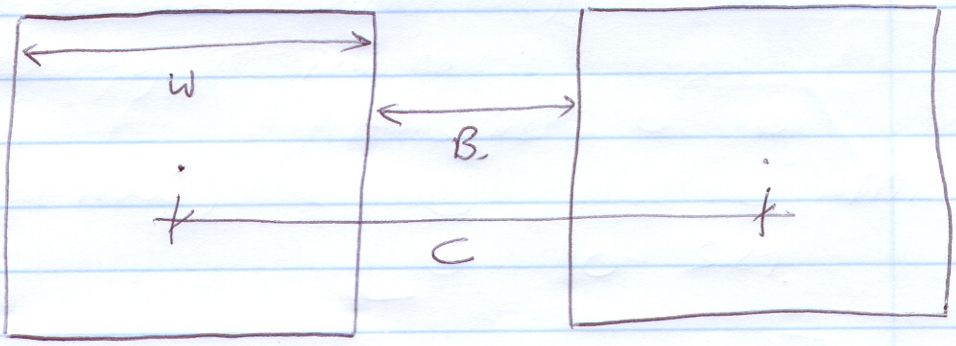
Question 1

Textbook questions.

Question 2

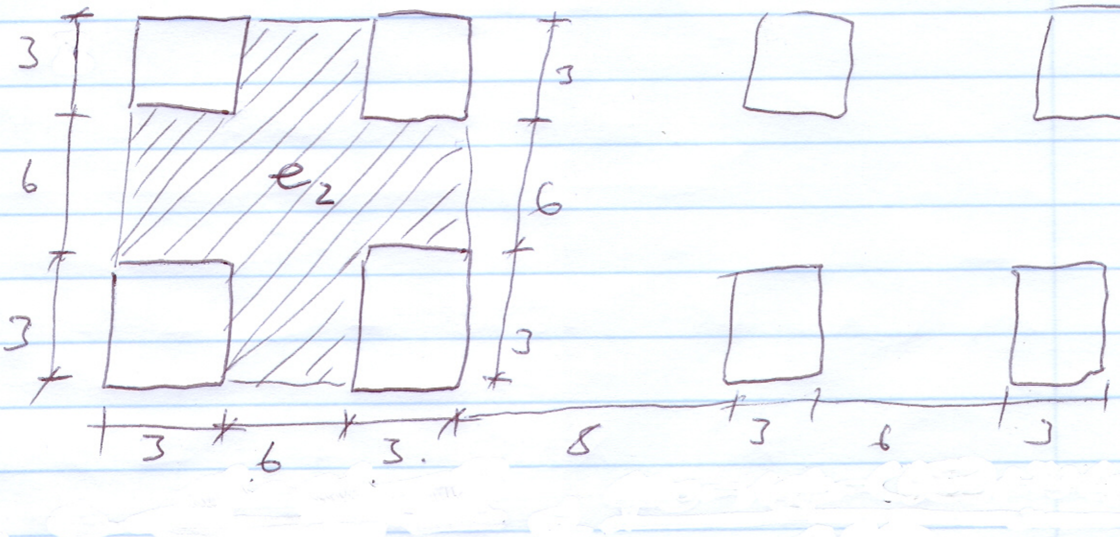
2.1

Before Secondary extraction



$$e = \frac{c^2 - w^2}{c^2} = \frac{20^2 - 12^2}{20^2} = 0.64$$
$$= 64\%. \quad 3$$

After Secondary Extraction



The second pass removes the shaded area marked e_2, leaving four pillars with area

$$4 \times 3 \times 3 = 36 m^2 = P$$

$$e = \frac{C^2 - P}{C^2} = \frac{400 - 36}{400}$$

$$= 0.91 \Rightarrow 91\%$$ 3

6.

2.2 Assume that in the central block, that span exceeds depth, so standard TAT can be use for pillars in the centre of the block.

After first pass:-

Pillar load $= \dfrac{WZ}{1-e} = \dfrac{29000 \times 320}{0.36}$

$$= 25.78\ MPa$$

Pillar Strength $= 0.3\ UCS \left(\dfrac{w^{0.5}}{h^{0.75}} \right)$

$$= 0.3 \times 360 \left(\frac{12^{0.5}}{5^{0.75}} \right) MPa$$

$$= 111.89\ MPa$$

$$FOS = \frac{\text{Strength}}{\text{load}} = \frac{111.89}{25.78} = 4.34$$ 2

After second pass:-

Pillar load $= \dfrac{WZ}{1-e} = 103.11\ MPa$

RMP1 - 3.

$$\text{Piller Strength} = 0.3 \times \cancel{UCS}\left(\frac{w^{0.5}}{h^{0.5}}\right)$$

$$= 0.3 \times 360 \left(\frac{3^{0.5}}{5^{0.75}}\right) \text{ MPa}$$

$$= \underline{55.94 \text{ MPa}}$$

$$\text{FOS} = \frac{\text{Strength}}{\text{load}} = \frac{55.94}{103.11} = \underline{0.54}$$ 2

According to the piller formula given, the remaining 3x3 m pillers in 6 x 8 m boards should have failed. The reason they have not is probably because the overlying strata are able to span the inner area mined, and therefore the TAT is not valid. More accurate numerical modelling is required to assess the situation. 2

6.

2.3 There is no way to evaluate the 3x3m piller geometry by hand, a numerical model will be necessary for this. The pillers will tend towards TAT loading as the mining using the 8x8 m pillers and 8 m bords extends beyond the old mined area. However, it is expected that the 8x8 m piller geometry will be stable, and the strata overlying the 3x3 m pillers will still be able to span the central block, loading the 3x3m pillers by less than their strength (ie. FOS for central area is probably actually greater than 1.0).

RMM - 4.

2.4 $e = \frac{C^2 - W^2}{C^2} = \frac{16^2 - 8^2}{16^2}$

$= 0.75 = 75\%$ ①

Pillar load $= \frac{Wz}{1-e} = \frac{29000 \times 320}{1 - 0.75}$

$= 37.12$ MPa

Pillar Strength $= 0.3\,UCS \left(\frac{W^{0.5}}{h^{0.75}}\right)$ MPa

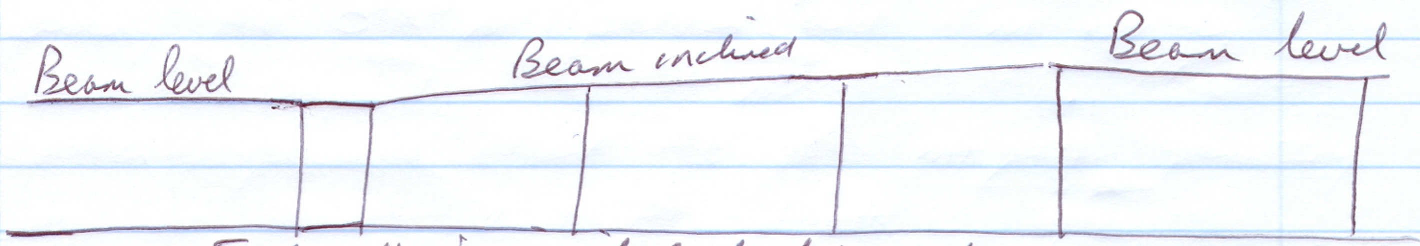
$= 0.3 \times 360 \left(\frac{8^{0.5}}{5^{0.75}}\right)$ MPa

$= 91.36$ MPa

FOS = 2.46 ③ Area will be stable. 4

2.5

(a) We will assume that the beam lies across the pillars as follows:-



(b) Assume also that the strength of the 3x3m pillars is not exceeded ie. they are still elastic.

(c) Assume that the footwall does not move (ie rigid)

RMP 5. (5)

(d) Assume horizontal stress is equal to vertical stress in virgin conditions.

(e) Assume pillars have only vertical stress acting in them, and that horizontal stress is zero.

(f) Assume Poisson's ratio = 0.3.

Then, the strain induced by the mining in the 3m pillar is given by

$$\Delta \varepsilon = \frac{1}{E}\left(\Delta\sigma_v - \nu\left(\Delta\sigma_{h_1} + \Delta\sigma_{h_2}\right)\right)$$

$$= \frac{1}{60\,000}\left(103.11 - 0.029 \times 320 - 0.3\left(0.029 \times 320 + 0.029 \times 320\right)\right)$$

$$= 0.001471$$

The strain in the second row of 8x8m pillars is considered as would be for TAT:

$$\Delta \varepsilon = \frac{1}{E}\left(\Delta\sigma_v - \nu\left(\Delta\sigma_{h_1} + \Delta\sigma_{h_2}\right)\right)$$

$$= \frac{1}{60\,000}\left(37.12 - 9.28 - 0.3\left(9.28 + 9.28\right)\right)$$

$$= 0.000371.$$

Assume that the strain in the row of 8m pillars is then halfway between that of the 3m pillar and the strain calculated for the 8m pillars above. Then...

$$\Delta \varepsilon = 0.000920.$$

Comments

The 8m pillars nearest to the 3m pillars experience an induced strain of nearly 2.4 times that of the next row of pillars. Therefore they will be more highly stressed and their factors of safety will therefore be lower. The lowest FOS is for the 3m pillars, and the highest FOS is 2.46 for the second and succeeding rows of 8 x 8 m pillars. The distribution of stress is probably more evenly spread over several rows of pillars. A numerical model is needed to obtain a better picture of the stress distribution in the pillars.

6

Question 3

3.1 When jacking the raisebore sidewalls back to their original positions, the virgin stress state will be measured (even though you are removing an induced strain in the rock by jacking the walls to their original positions).

6

3.2 $F_1 = 1500\ kN$, $\sigma_1 = \underline{1.5\ MPa}$

$F_2 = 800\ kN$ $\sigma_2 = \underline{0.8\ MPa}$

6

3.3 $\sigma_v = wz = 25000 \times 30 = 750\ kPa$

$= \underline{0.75\ MPa}$

(We assume unit weight of sandstone is $25\ kN/m^3$).

I consider the results reasonable because both the horizontal stresses are greater than the vertical virgin stress as was already known. Secondly the horizontal virgin stresses are unequal as was also known. We do not have any better data than that found, so we must accept our results as being reasonable – at least they do not contradict our expectations.

5

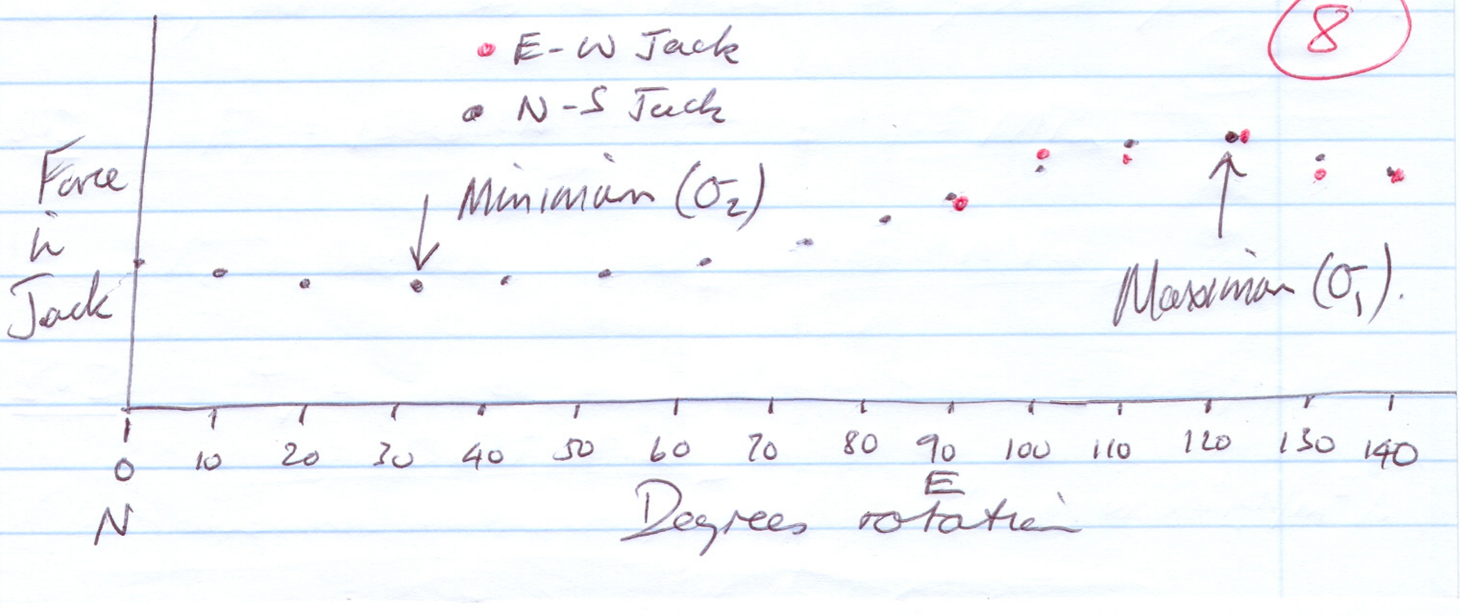
3.4. (1) Start with jacks in N-S, and E-W position, jack out to original diameter and take force readings.

(2) Rotate N-S jack to 10°E of N and E-W jack 10° S of E, and jack out to original diameter and take force readings.

(3) Repeat (2) at 10° intervals until there is a clear maximum and minimum force reading in both sets of jacks.

The data will probably look like this:–

8



Question 4

4.1

Assume that the solid material is a right cylinder with parallel ends. To keep volumetric calculations simple, assume that there is no friction between the sample and the machine platens (so that sample expands uniformly in the horizontal direction)

$$V_0 = 2\pi r^2 l.$$

$$V_0 + \Delta V = 2\pi (r+\Delta r)^2 (l + \Delta l)$$

expanding

$$V_0 + \Delta V = 2\pi r^2 l + 2\pi \left(2\Delta r\, r l + \Delta r^2 l + \Delta r^2 \Delta l + 2\Delta r \Delta l\, r + \Delta r^2 \Delta l\right).$$

$$\Rightarrow \Delta V = 2\pi \left(2\Delta r\, r l + \Delta r^2 l + r^2 \Delta l + 2\Delta r \Delta l\, r + \Delta r^2 \Delta l\right). \quad (1)$$

We have not discarded the second order terms deliberately. In terms of strain (here we will use the "incorrect" definition of strain) a uniaxial test produces the following strains:-

$$\varepsilon_1 = \frac{\Delta l}{l_0}, \quad \varepsilon_2 = \varepsilon_3 = \frac{\Delta r}{r_0}; \text{ and } \varepsilon_2 + \varepsilon_3$$

are uniform throughout the full length of the sample (this simplifying assumption comes from assuming a frictionless contact between cylinder and machine platens).

Therefore,

$$\Delta l = \varepsilon_1 l_0 = \varepsilon_1 l$$

$$\Delta r = \varepsilon_2 r = \varepsilon_3 r$$

Substituting this into equation 1 we have

$$\Delta V = 2\pi\left((\varepsilon_2+\varepsilon_3)r^2 l + \varepsilon_2\varepsilon_3 r^2 l + \varepsilon_1 r^2 l + (\varepsilon_2+\varepsilon_3)\varepsilon_1 r^2 l + \varepsilon_1\varepsilon_2\varepsilon_3 r^2 l\right)$$

$$= 2\pi r^2 l\left(\varepsilon_2+\varepsilon_3+\varepsilon_2\varepsilon_3+\varepsilon_1+\varepsilon_1\varepsilon_2+\varepsilon_1\varepsilon_3 + \varepsilon_1\varepsilon_2\varepsilon_3\right). \quad \text{— } 2$$

By definition, volumetric strain is:-

$$\varepsilon_V = \frac{\Delta V}{V} = \frac{2\pi r^2 l}{2\pi r^2 l}\left(\varepsilon_1+\varepsilon_2+\varepsilon_3+\varepsilon_1\varepsilon_2+\varepsilon_2\varepsilon_3+\varepsilon_1\varepsilon_3+\varepsilon_1\varepsilon_2\varepsilon_3\right)$$

or,

$$\varepsilon_V = \left(\varepsilon_1+\varepsilon_2+\varepsilon_3+\varepsilon_1\varepsilon_2+\varepsilon_2\varepsilon_3+\varepsilon_1\varepsilon_3+\varepsilon_1\varepsilon_2\varepsilon_3\right).$$

If ε_1, ε_2 and ε_3 are "small", i.e. of the order of $\frac{1}{1000}$, then the second order terms $\varepsilon_1\varepsilon_2$ etc are of the order of $\frac{1}{1000\,000}$, and the third order term $\varepsilon_1 \varepsilon_2 \varepsilon_3$ is of the order of $\frac{1}{1000\,000\,000}$.

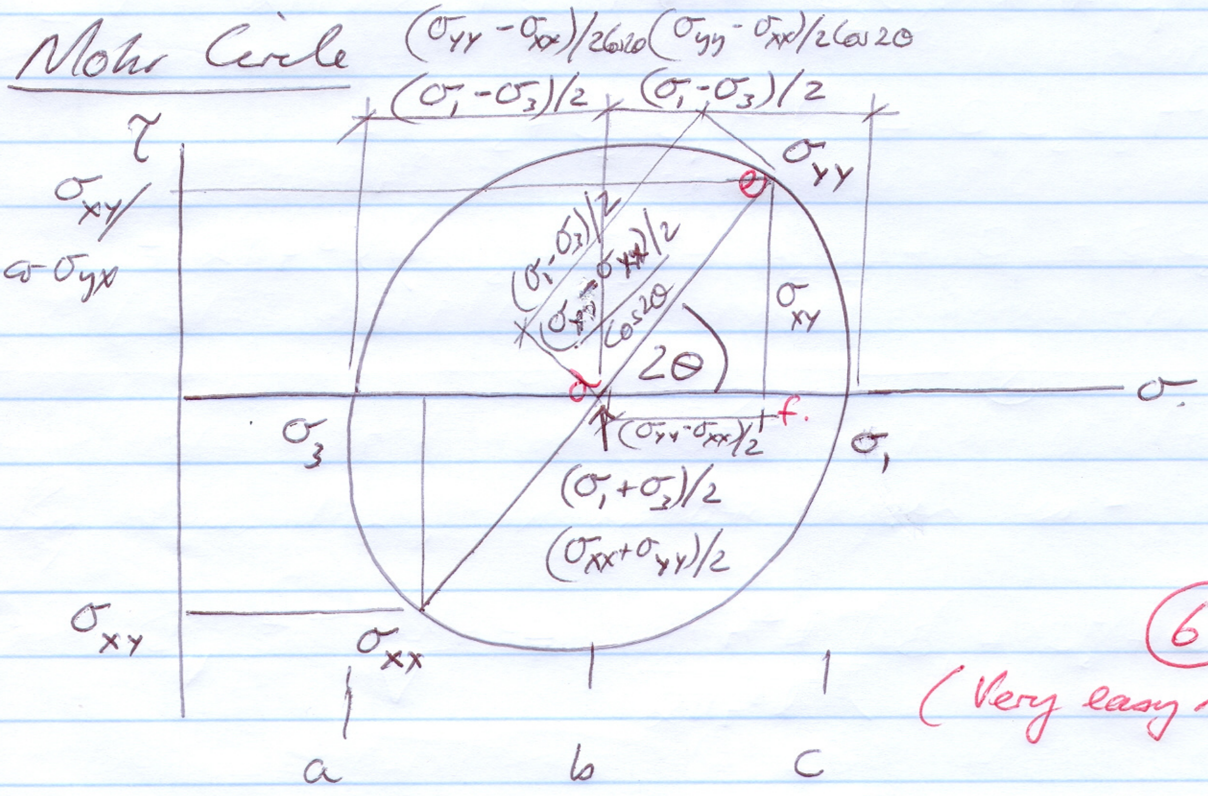
The second and third order terms may thus be neglected and

$$\varepsilon_V \cong \varepsilon_1 + \varepsilon_2 + \varepsilon_3 .$$ qed

6 (6 hard marks)

4.2

Mohr Circle



6 (Very easy marks)

4.3

$(de)^2 = df^2 + ef^2$

$= \frac{1}{4}(\sigma_{yy}-\sigma_{xx})^2 + \sigma_{xy}^2$

$= \frac{1}{4}\left[(\sigma_{yy}-\sigma_{xx})^2 + 4\sigma_{xy}^2\right]$

$de = \frac{1}{2}\sqrt{\ldots}$

From the circle above

$$\sigma_1 = \frac{1}{2}(\sigma_{yy}+\sigma_{xx}) + \frac{1}{2}\sqrt{(\sigma_{yy}-\sigma_{xx})^2 + 4\sigma_{xy}^2}$$

Similarly

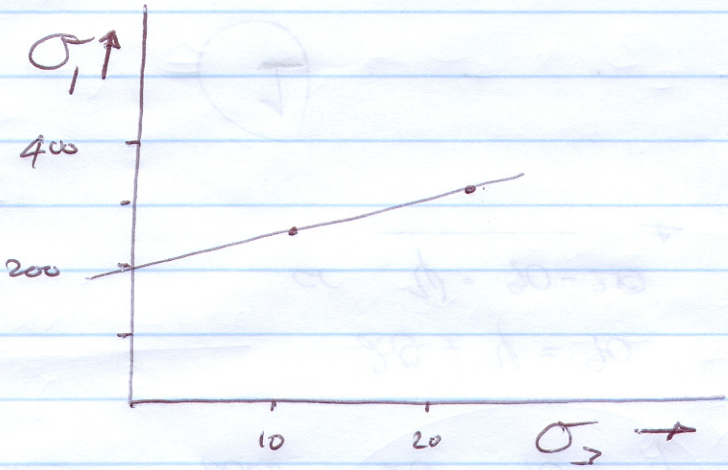
$$\sigma_3 = \frac{1}{2}(\sigma_{yy}+\sigma_{xy}) - \frac{1}{2}\sqrt{(\sigma_{yy}-\sigma_{xx})^2 + 4\sigma_{xy}^2}$$

qed

6 easy marks.

4.4 $\sigma_1 = 320$ MPa, $\sigma_3 = 20$ MPa failure

$\sigma_1 = 260$ MPa, $\sigma_3 = 10$ MPa failure



The angle of dilation has no bearing on this problem. It is a deliberate red herring.

If the relationship is linear, then

$$\sigma_1 = \sigma_c + b\sigma_3.$$

$$b = \frac{320-260}{20-10} = 6.$$

$$\therefore \quad \sigma_1 = \sigma_c + 6\sigma_3.$$

Using values of $\sigma_1 = 320$ MPa, and $\sigma_3 = 20$ MPa:

$$320 = \sigma_c + 6 \times 20$$

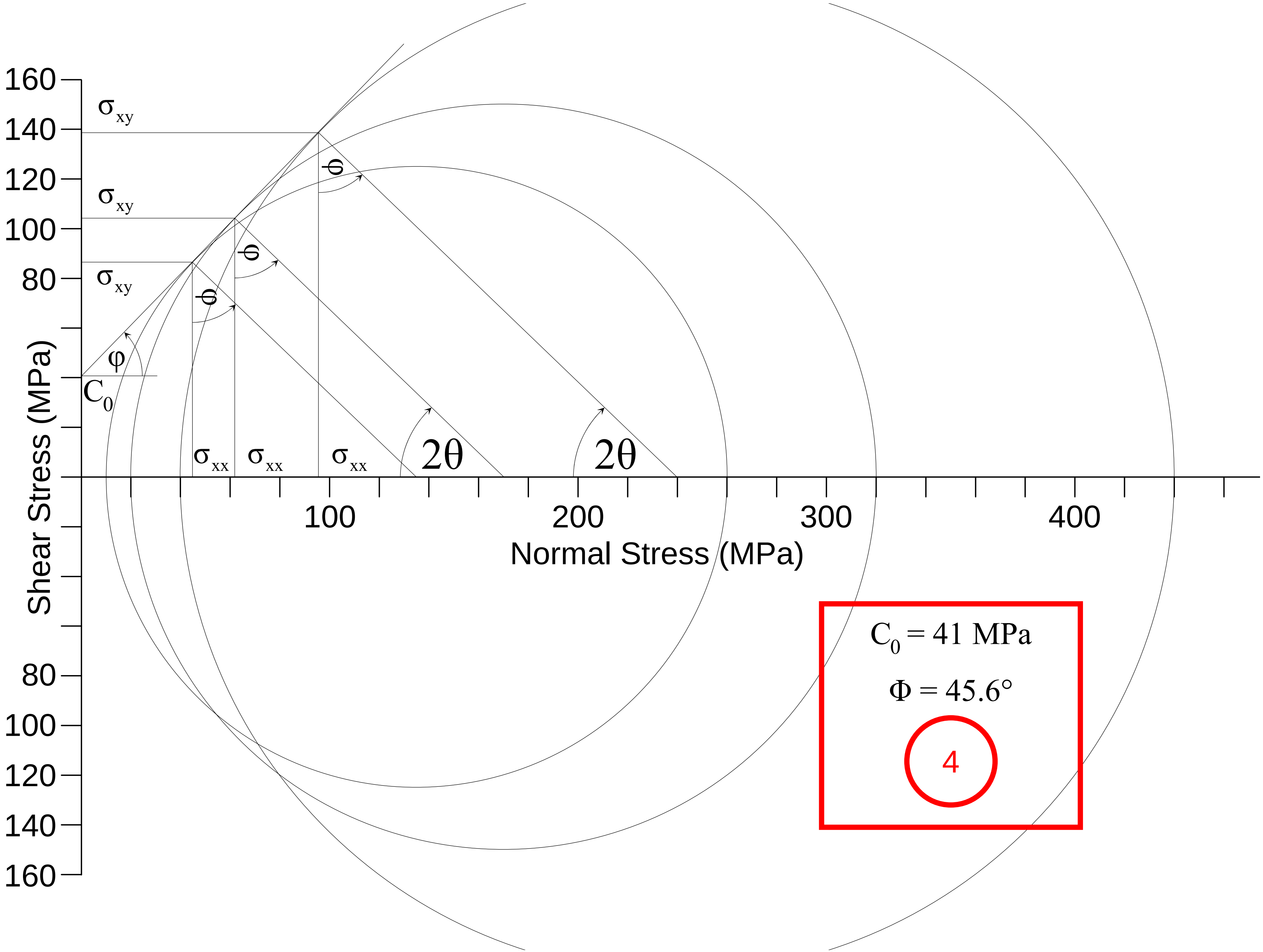
$$\Rightarrow \quad \sigma_c = 200 \text{ MPa}$$

3.

4.5

One has to plot a Mohr Circle, and find a failure envelope, or solve analytically.

PTO.



160
140
120
100
80
80
100
120
140
160
Shear Stress (MPa)
100
200
300
400
Normal Stress (MPa)
σxy
σxy
σxy
σxx
σxx
σxx
φ
C0
2θ
2θ
C0 = 41 MPa
Φ = 45.6°
4

The graphical Solution is:-

$$\boxed{\begin{aligned} C_o &= 41 \text{ MPa} \\ \phi &= 45.55^\circ \end{aligned}}$$

See attached graph.

This is not easy to get graphically because the two Mohr Circles plot very close to each other and it is difficult to estimate proper position of tangent to both.

The only reason the above results are so close to the analytical result is because I plotted 3 Mohr Circle very accurately in Powerpoint and was able to read off tangent accurately.

The analytical Solution is:

Measurement	σ_3	σ_1	$\frac{1}{2}(\sigma_1+\sigma_3)$	$\frac{1}{2}(\sigma_1-\sigma_3)$
1	10	260	135	125
2	20	320	170	150

$$\cot\phi = \frac{\Delta x}{\Delta y} = \frac{170 - 150\sin\phi - 135 + 125\sin\phi}{150\cos\phi - 125\cos\phi}$$

$$25\,\frac{\cos\phi}{\sin\phi}\cos\phi = 35 - 25\sin\phi$$

$$25\cos^2\phi + 25\sin^2\phi = 35\sin\phi$$

$$\sin\phi = \frac{25}{35} = 0.714286$$

$$\phi = 45.58^\circ$$

13.

The Cohesion is given by :-

$$|\sigma_{xr(i)}| = C_0 + \sigma_{xx(i)} \tan\phi.$$

$$\Rightarrow C_0 = 150\cos\phi - [170 - 150\sin\phi]\tan\phi$$

$$= 104.98 - 64.15$$

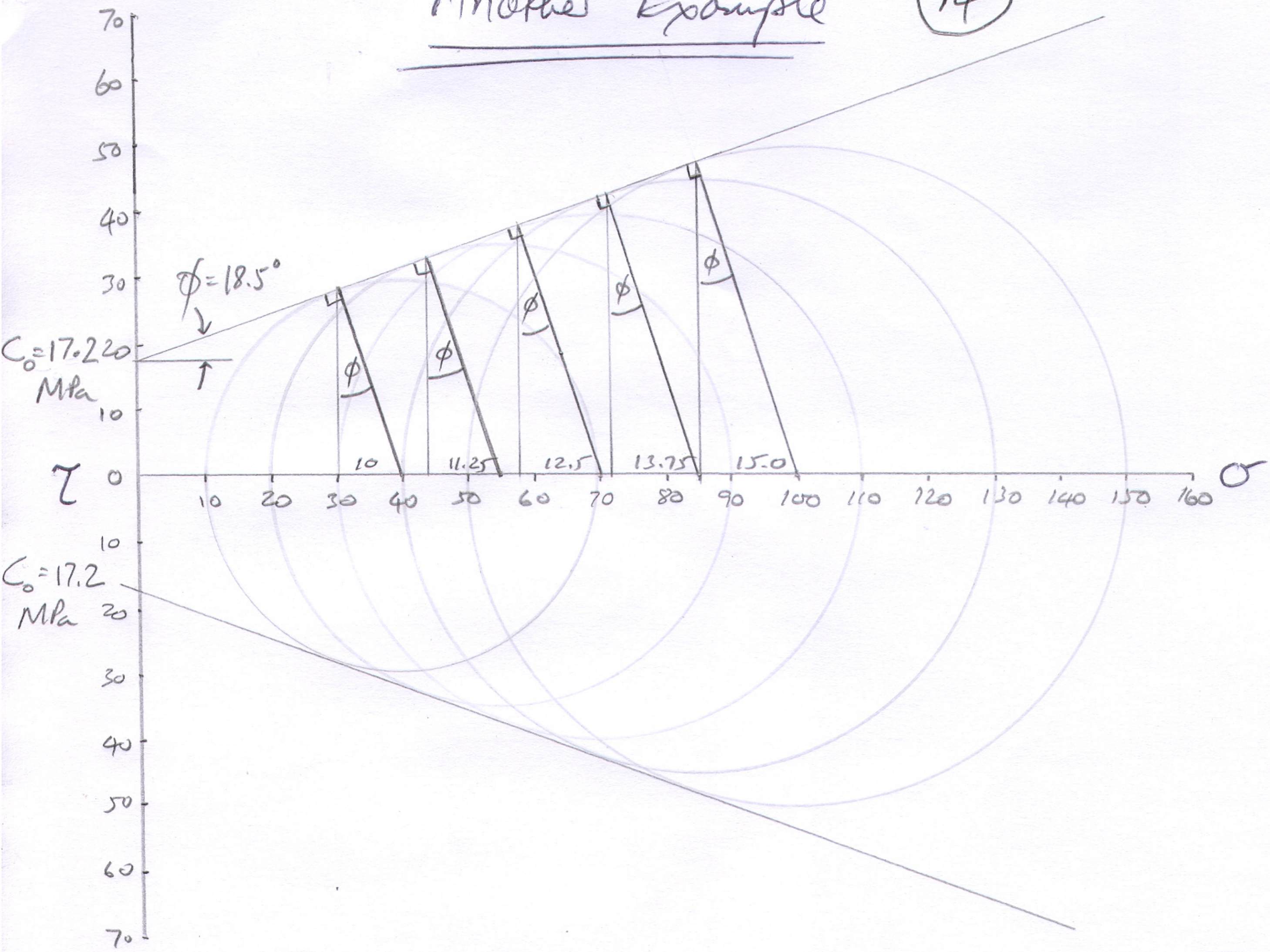
$$= 40.82 \text{ MPa}$$

4

25

for whole question.

Another Example (14)



Use relationship $\sigma_1 = \sigma_c + 2\sigma_3$ (to make envelope flatter). Let $\sigma_c = 50$ MPa

σ_{xx}	$\frac{1}{2}(\sigma_1-\sigma_3)\sin\phi$	$\frac{1}{2}(\sigma_1-\sigma_3)\cos\phi$	$\sigma_3 =$	$= \sigma_1$	$\frac{1}{2}\sigma_1+\sigma_3$	$\frac{1}{2}\sigma_1-\sigma_3$
30.48	9.52	28.45	10	70	40	30
43.89	11.11	33.19	20	90	55	35
57.31	12.69	37.93	30	110	70	40
70.72	14.28	42.67	40	130	85	45
84.13	15.87	47.42	50	150	100	50.

$$\sigma_{xx(i)} = \frac{1}{2}(\sigma_1+\sigma_3)_i - \frac{1}{2}(\sigma_1-\sigma_3)_i \sin\phi.$$

$$\sigma_{xy(i)} = \frac{1}{2}(\sigma_1-\sigma_3)_i \cos\phi.$$

and.. $|\sigma_{xy}|_i = C_o + \sigma_{xx(i)} \tan\phi.$

15 Pure Analytical

Solving for ϕ: (Here I have used what I got for ϕ from graph)

$$\tan\phi = \frac{\sigma_{xx(1)} - \sigma_{xx(2)}}{\sigma_{xy(1)} - \sigma_{xy(2)}}$$

$$= \left(\frac{84.13 - 30.48}{47.42 - 28.45}\right)^{-1}$$

$$= 0.353588$$

$$\phi = 19.47^\circ$$

$$\cot\phi = \left(100 - 50\ \ \phi - 40 + 30\ \ \phi\right) / \left(50\cos\phi - 30\cos\phi\right)$$

$$\frac{20\cos^2\phi}{\sin\phi} = 60 - 20\sin\phi$$

or $\frac{20}{60} = \sin\phi \Rightarrow \phi = 19.47^\circ$

$$C_o = |\sigma_{xy(1)}| - \sigma_{xx(1)}\tan\phi$$

$$= 47.42 - 84.13\tan 19.47$$

$$= 17.67\ MPa$$

$$C_o = |\sigma_{xy(2)}| - \sigma_{xx(2)}\tan\phi$$

$$= 28.28 - 10.61$$

$$= 17.68\ MPa$$

V. Close to what I have above!